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Income Redistribution Systems Financed by Import Tariffs

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Abstract

We build a simple two-country monopolistic competition trade model in which there are two types of agents in each country (home and foreign): a high-income agent and a low-income agent. We compare the social welfare effects of two tax-transfer systems in the model: one system is financed by income taxes and the other is financed by import tariffs. Both tax-transfer systems improve domestic equality by raising utility levels of low-income individuals and reducing those of high-income individuals. However, equality improvement costs—represented by the scale of the deterioration in high-income individuals' utility needed to improve low-income individuals' utility—are smaller under a tax-transfer system financed by import tariffs than simple linear income taxes. The results show that the properties of import tariffs are similar to progressive income taxes.

Keywords: income inequality, income redistribution systems; import tariffs, income tax

JEL classification codes: D31; D63; F12; F13

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1. Introduction

The rise in income inequality in many countries with the expansion of international trade has caused social controversy over globalization.[1] Dissatisfaction due to rising income inequality could increase the anti-globalization drift in society. The proponents of trade liberalization insist that a proper income redistribution policy—not a protective trade policy—is required to address rising domestic income inequality.[2]

Most international economists prefer trade liberalization, despite its distributional conflicts, because they evaluate the social welfare effects of trade liberalization according to the Kaldor–Hicks compensation principle (Hicks, 1939; Kaldor, 1939). This approach judges the "gains from trade" from the potential feasibility of Pareto improvement—that trade liberalization can create a surplus large enough to compensate the losers. Dixit and Norman (1980, 1986) propose a system of commodity taxes and subsidies that would allow trade liberalization to meet the compensation principle in a setting of perfect competition markets and a small open economy.

In recent years, several studies have examined the link between trade liberalization and the income redistribution system in the literature on the heterogeneous firms/agents trade model introduced by Melitz (2003). Itskhoki (2008) presents a model that integrates Mirrlees' (1971) optimal taxation framework with a heterogeneous agents trade model. The researcher shows that a tax-transfer system financed by income taxes improves social welfare by reducing income inequality, although globalization progressively expands income inequality between high-ability agents engaged in exports and low-ability agents not engaged in exports. Egger and Kreickemier (2009) integrate a

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[1] Several empirical studies show that trade liberalization is robustly related to increased income inequality in developed countries (Bergh and Nilsson, 2010; Dreher and Gaston, 2008). Goldberg and Pavcnik (2007) investigate the distributional effects of globalization in developing countries during the 1980s and 1990s, by presenting cases in which trade liberalization triggered significant rise in income inequality.

[2] Irwin (2008, pp. 142–143) insists that "inequality may be undesirable, but it should be addressed not by closing markets through greater protectionism, but by more progressive income taxation, a stronger social safety net, and more assistance for displaced workers."

tax-transfer system into a trade model with heterogeneous firms and fair wages. They show that trade liberalization can lower income inequality and increase aggregate income under autarky when it is accompanied by a tax-transfer system financed by profit taxes. Kohl (2020) presents a trade model that accompanies occupational choice between managers and workers with a redistribution scheme financed by progressive income tax to expose the conditions under which trade leads to Pareto improvement.

These past studies commonly assume a welfare state that redistributes income from a high-income group to a low-income group by uniformly transferring the tax revenues levied on income or profit to all individuals. They show that the redistribution system is accompanied by a trade-off between equality and efficiency: It reduces income inequality at the cost of reducing aggregate income. Therefore, whether the income redistribution system improves social welfare depends on the extent to which a society allows aggregate income reduction in exchange for a reduction in income inequality.

In this study, we present another type of income redistribution scheme: a tax-transfer system financed by import tariffs.[3] We build a simple two-country monopolistic competition trade model in which there are two types of agents in each country: a high-income agent and a low-income agent. A high-income agent has high ability and can export their products abroad, while a low-income agent has low ability and supplies their products only to the domestic market. An inequality-averse government can improve social welfare through an income redistribution system financed either by income taxes or import tariffs. Our model shows that both redistribution systems have an income transfer effect that increases the real revenues of low-income agents while reducing high-income

[3] This study is not the first to address an income redistribution system based on import tariffs. Naito (1996) and Saez (2004) show that, if a government can use a redistributive non-linear income tax system as well as tariffs, it can realize Pareto improvement in welfare by using an import tariff policy.

agents' revenues. This reduces the utility gap between high- and low-income agents and improves social welfare when a government's inequality-averse preferences are sufficiently strong.

We compare the efficiency of redistribution systems financed by income and import tariffs. We define the cost of improving equality as the scale of utility deterioration of high-income agents per unit of utility improvement for low-income agents. By comparing equality improvement costs of redistribution systems financed by income taxes and import tariffs, we reveal that import tariffs can be a more efficient financial resource for a tax-transfer system than income tax. In particular, we show that when only one of the two countries introduces a redistribution system financed by import tariffs, that country can not only reduce utility inequality but also increase aggregate income and utility.

It is well known that, in the two-country trade model, import tariffs increase aggregate real income in the country that implements the policy.[4] Conversely, some studies insist that import tariffs are "beggar-thy-neighbor" policies that reduce aggregate real income in the foreign country, and that the import tariff policy creates income losses when the trade partner country implements the same policy. In contrast, our model shows that rising import tariffs in both countries can improve social welfare by improving domestic inequality, although it does reduce real aggregate income.

The remainder of this paper proceeds as follows. Section 2 introduces the model. In section 3, we explore the economic effects of an income redistribution scheme financed by a simple linear income tax. In section 4, we analyze the effects of the redistribution scheme financed by the import tariff and compare the equality improvement costs of the tax-transfer system financed by the income tax and import tariff. In section 5, we analyze the effects of the redistribution scheme financed by the income tax levied only on high-income agents. From this, we reveal that the import tariff has characteristics similar to those of progressive income tax. Finally, section 6 concludes the paper.

[4] For a monopolistic competition trade model, see Gros (1987) and Felbermayr et al. (2013).

2. Model setup

We consider a setup of two countries—home and foreign—populated by a measure L (L^*) of worker–entrepreneurs.[5] Each agent produces a variety of differentiated final goods based on their own labor. Agents are divided into two groups: Group H and Group N. While an agent in Group N (N-agent) supplies their variety to only to the domestic market, an agent in Group H (H-agent) has a higher level of productivity than the N-agent and exports their variety without incurring any transportation or fixed costs.[6]

The production function of the j-agent (j=H, N) is represented as

$$y_j = n_j l_j, \quad (j=H,N), \tag{1}$$

where y_j is the output variety, l_j is the labor input, and n_j is the ability of the j-agents ($n_H \geq n_N$). The H-agent supplies y_{Hd} out of their output y_H to the domestic market and exports y_{He}.

We adopt the Greenwood–Hercowitz–Huffman preference (Greenwood et al., 1988) for each agent as follows:

$$u_j = c_j - \frac{1}{\gamma} l_j{}^{\gamma}, \gamma = 1 + \frac{1}{\varepsilon}, \tag{2}$$

where c_j represents final goods consumption, which is equal to real disposable income, while the second term on the right-hand side of Eq. (2) represents the disutility suffered from inputting labor into the production activity. ε is the constant labor supply elasticity. In this study, we assume that labor supply elasticity is sufficiently small, that is, $\varepsilon<1$ (i.e., $\gamma>2$).[7]

As for the consumption of final goods, we assume that all agents have constant elasticity of substitution (CES) preferences, as in Dixit and Stiglitz (1977). Thus, the aggregate real consumption in the home country Q is represented by

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[5] All foreign variables are denoted with an asterisk.

[6] For tractability, we assume no transportation costs for trade and no fixed costs for starting trade. Thus, our model is similar to Helpman and Krugman's (1985) classical monopolistic competition trade model, rather than to the Melitz-type model.

[7] After reviewing the literature on estimating labor supply elasticity, Whalen and Reichling (2017) find that labor supply elasticity estimates range between 0.27 and 0.53.

$$Q = \left[\int_0^{L(1-\chi)} y_N(\omega)^\beta d\omega + \int_{L(1-\chi)}^{L} y_{Hd}(\omega)^\beta d\omega + \int_{L^*(1-\chi^*)}^{L^*} y_{He}{}^*(\omega)^\beta d\omega\right]^{\frac{1}{\beta}}, 0\leq\beta\leq1 \quad (3)$$

where y_{He}* is the consumption of the import variety, 1/(1-β)>1 is the elasticity of substitution between different varieties, and χ (χ*) represents H-agents' population share in the home (foreign) country. Note that the number of varieties consumed in the home country is the sum of domestic agents and H-agents abroad. Q* is defined in Eq. (3). The price index associated with the final goods consumption is obtained as follows:[8]

$$P = \left[\int_0^{L(1-\chi)} p_N(\omega)^{-\frac{\beta}{1-\beta}} d\omega + \int_{L(1-\chi)}^{L} p_{Hd}(\omega)^{-\frac{\beta}{1-\beta}} d\omega + \int_{L^*(1-\chi^*)}^{L^*} \left((1+T)p_{He}{}^*(\omega)\right)^{-\frac{\beta}{1-\beta}} d\omega\right]^{-\frac{1-\beta}{\beta}}, \quad (4)$$

where p represents the price of a variety, and T represents a uniform ad valorem tariff on import varieties imposed by the home government.

From the CES aggregators Q and Q*, domestic and export demand for varieties produced by the home agents are represented by

$$y_N = \left(\frac{p_N}{P}\right)^{-\frac{1}{1-\beta}} Q,\ y_{Hd} = \left(\frac{p_{Hd}}{P}\right)^{-\frac{1}{1-\beta}} Q,\ y_{He} = \left(\frac{(1+T^*)p_{He}}{P^*}\right)^{-\frac{1}{1-\beta}} Q. \quad (5)$$

From Eq. (5), the N-agent's real revenue r_N is

$$r_N = y_N{}^\beta Q^{1-\beta}. \quad (6)$$

The H-agent allocates their output y_H to domestic sales y_{Hd} and exports y_{He} to maximize their real revenue as follows:

$$r_H = \max_{\substack{y_{Hd}, y_{He} \\ y_{Hd}+y_{He}=y_H}} \left\{\frac{y_{Hd}{}^\beta PQ^{1-\beta} + y_{He}{}^\beta P^* Q^{*1-\beta}}{P}\right\} = y_H{}^\beta PQ^{1-\beta}(1+Y_x)^{1-\beta}. \quad (7)$$

Here, Y_x is the ratio of export volume to domestic supply, represented as

$$Y_x = \frac{y_{He}}{y_{Hd}} = (1+T^*)^{-\frac{1}{1-\beta}} \left(\frac{P^*}{P}\right)^{\frac{\beta}{1-\beta}} \left(\frac{P^*Q^*}{PQ}\right). \quad (8)$$

Since Y_x* is defined as Y_x, the relationship between Y_x and Y_x* is represented as

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[8] Since the total nominal expenditure on final goods is represented by PQ, Q represents total real expenditure on final goods.

$$Y_x Y_x{}^* = (1+T^*)^{-\frac{1}{1-\beta}}(1+T)^{-\frac{1}{1-\beta}}. \tag{9}$$

The government affects agents' real disposable income through the income redistribution system. The government collects tax revenue and redistributes it uniformly to all agents in the economy. Agents' disposable income, that is, *post-transfer* income, is the sum of the market (*pre-transfer*) income and net transfer income obtained from the government.

In this study, we compare two schemes in the income redistribution system: the income tax scheme financed by income tax and the import tariff scheme financed by import tariffs. Taking the government's budget constraint into consideration, *j*-agent's real disposable income is represented

$$c_j = \begin{cases} r_j + t\left(\frac{R}{L} - r_j\right) & (Income\ Tax\ Scheme) \\ r_j + \frac{TA}{L} & (Import\ Tariff\ Scheme) \end{cases}. \tag{10}$$

where R represents aggregate real revenue in the home country (i.e., $R=(1-\chi)Lr_N+\chi Lr_H$), and t is the uniform income tax rate. TA represents real tariff revenue. In the income tax scheme, when the H-agent's real revenue r_H is larger than the N-agent's real revenue r_N, and the N-agent (H-agent) is the net transfer receiver (tax payer), the sign of the second term on the right-hand side of Eq. (10) is positive (negative).

Since an agent manages their labor input to maximize their utility, from Eqs. (1), (2), (5), (6), (7), and (10), the output of a variety, the real revenue, and the maximized utility of agents in the home country are obtained as

$$y_N = \beta^{\frac{1}{\gamma-\beta}}(1-t)^{\frac{1}{\gamma-\beta}}Q^{\frac{1-\beta}{\gamma-\beta}}n_N{}^{\frac{\gamma}{\gamma-\beta}},\ y_H = \beta^{\frac{1}{\gamma-\beta}}(1-t)^{\frac{1}{\gamma-\beta}}Q^{\frac{1-\beta}{\gamma-\beta}}(1+Y_x)^{\frac{1-\beta}{\gamma-\beta}}n_H{}^{\frac{\gamma}{\gamma-\beta}}, \tag{11}$$

$$r_N = \beta^{\frac{\beta}{\gamma-\beta}}(1-t)^{\frac{\beta}{\gamma-\beta}}Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}}n_N{}^{\frac{\beta\gamma}{\gamma-\beta}},\ r_H = \beta^{\frac{\beta}{\gamma-\beta}}(1-t)^{\frac{\beta}{\gamma-\beta}}Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}}(1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}}n_H{}^{\frac{\beta\gamma}{\gamma-\beta}}, \tag{12}$$

$$u_j = \begin{cases} \left(1-\frac{\beta}{\gamma}\right)(1-t)r_j + \frac{tR}{L} & (Income\ Tax\ Scheme) \\ \left(1-\frac{\beta}{\gamma}\right)r_j + \frac{TA}{L} & (Import\ Tariff\ Scheme) \end{cases}. \tag{13}$$

Eqs. (12) and (13) imply that the H-agent obtains higher revenue and utility than the N-agent (i.e., $r_H>r_N$, $u_H>u_N$).

3. The income tax scheme

In this section, we analyze the income redistribution system based on the income tax scheme. The first part presents an equilibrium in an open economy and a comparative statics system to analyze the economic impact of income tax. In the second part, we analyze the social welfare effects of income tax when only the home government raises its income tax. In the third part, we analyze the case where both the home and foreign governments raise their income tax. To simplify our analysis, we assume the home and foreign countries are completely symmetrical (i.e., $L=L^*$, $\chi=\chi^*$, $n_N=n_N^*$, and $n_H=n_H^*$), although our results are not restricted to the symmetric case.

3.1. Trade equilibrium and comparative statics system

When the home and foreign governments follow a free trade policy (i.e., $T=T^*=0$), balanced trade implies that the export value of the home country, $\chi L p_{He} y_{He}$, always equals that of the foreign country, $\chi^* L^* p_{He}^* y_{He}^*$. Thus, from Eqs. (5), (8), and (11), we derive the trade balance condition as follows:

$$Y_x{}^{1-\beta} = \left(\frac{1-t^*}{1-t}\right)^{\frac{\beta}{\gamma-\beta}} \frac{\chi^* L^*}{\chi L} \left(\frac{Y_x{}^*}{Y_x}\right)^{\beta} \left(\frac{1+Y_x}{1+Y_x{}^*}\right)^{\frac{\beta(\gamma-1)}{\gamma-\beta}} \left(\frac{Q^*}{Q}\right)^{\frac{\beta(1-\beta)}{\gamma-\beta}} \left(\frac{n_H{}^*}{n_N}\right)^{\frac{\beta\gamma}{\gamma-\beta}}. \tag{14}$$

Aggregate real revenue in home country R is obtained from Eq. (12) as follows:

$$R = (1-\chi)Lr_N + \chi L r_H = \beta^{\frac{\beta}{\gamma-\beta}}(1-t)^{\frac{\beta}{\gamma-\beta}} Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}} \left((1-\chi)n_N{}^{\frac{\beta\gamma}{\gamma-\beta}} + \chi(1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}} n_H{}^{\frac{\beta\gamma}{\gamma-\beta}}\right) L. \tag{15}$$

Since balanced trade implies that aggregate real revenue R equals aggregate real consumption Q in the home country, we obtain the following equation from Eq. (15):

$$Q^{\frac{\beta(\gamma-1)}{\gamma-\beta}} = \beta^{\frac{\beta}{\gamma-\beta}}(1-t)^{\frac{\beta}{\gamma-\beta}}\left((1-\chi)n_N{}^{\frac{\beta\gamma}{\gamma-\beta}} + \chi(1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}} n_H{}^{\frac{\beta\gamma}{\gamma-\beta}}\right)L. \tag{16}$$

Similarly, we obtain the following equation for foreign aggregate real consumption Q^*:

$$Q^{*\frac{\beta(\gamma-1)}{\gamma-\beta}} = \beta^{\frac{\beta}{\gamma-\beta}}(1-t^*)^{\frac{\beta}{\gamma-\beta}}\left((1-\chi^*)n_N{}^{*\frac{\beta\gamma}{\gamma-\beta}} + (1+Y_x{}^*)^{\frac{\gamma(1-\beta)}{\gamma-\beta}} n_H{}^{*\frac{\beta\gamma}{\gamma-\beta}}\right)L^*. \tag{17}$$

The equilibrium values Q, Q^*, Y_x, and $Y_x{}^*$ under the redistributive tax policy are obtained from Eqs. (9), (14), (16), and (17). From these equations, we obtain the following comparative statics matrix:

$$\begin{bmatrix} a_{11} & -a_{11} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23}{}^{TX} & 0 \\ 0 & a_{21} & 0 & a_{34}{}^{TX} \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} dlnQ \\ dlnQ^* \\ dlnY_x \\ dlnY_x{}^* \end{bmatrix} = \begin{bmatrix} b_1{}^{TX} \\ b_2{}^{TX} \\ b_3{}^{TX} \\ 0 \end{bmatrix}, \tag{18}$$

where $a_{11} = \frac{\beta(1-\beta)}{\gamma-\beta}$, $a_{13} = 1-\beta+\beta\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e\right)$, $a_{14} = -\beta\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)$, $a_{21} = \frac{\beta(\gamma-1)}{\gamma-\beta}$,

$a_{23}{}^{TX} = -\frac{\gamma(1-\beta)}{\gamma-\beta}R_H Y_e$, $a_{34}{}^{TX} = -\frac{\gamma(1-\beta)}{\gamma-\beta}R_H{}^* Y_e{}^*$,

$b_1{}^{TX} = \frac{\beta}{\gamma-\beta}\left(dln(1-t^*) - dln(1-t)\right)$, $b_2{}^{TX} = \frac{\beta}{\gamma-\beta}dln(1-t)$, $b_3{}^{TX} = \frac{\beta}{\gamma-\beta}dln(1-t^*)$,

$Y_e = \frac{y_{He}}{y_H} = \frac{Y_x}{1+Y_x}$, $Y_e{}^* = \frac{y_{He}{}^*}{y_H{}^*} = \frac{Y_x{}^*}{1+Y_x{}^*}$, $R_H = \frac{\chi L r_H}{R}$, and $R_H{}^* = \frac{\chi^* L^* r_H{}^*}{R^*}$.

Y_e is the export to total output ratio of the H-agent. R_H is the H-agent's real revenue share relative to aggregate real revenue.

3.2. The case where only the home government implements the income tax scheme

When the foreign government has no policy (i.e., $t^*=0$), we obtain from the comparative statics matrix in Eq. (18) the elasticity of the aggregate real consumption (revenue) Q and the export volume to domestic supply ratio Y_x in the home country with respect to the marginal tax rate t as follows:[9]

[9] Note that $dln(1-t)<0$ when the marginal tax rate t increases.

$$-\frac{dlnQ}{dln(1-t)} = -\frac{\beta^2}{(\gamma-\beta)^2}\frac{1}{\Delta^{TX}}\begin{pmatrix}\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H{}^*Y_e{}^* + (\gamma-1)(1-\beta)\left(1-\frac{\gamma}{\gamma-\beta}R_HY_e\right)\\ +(\gamma-1)\beta\left(2-\frac{\gamma-1}{\gamma-\beta}Y_e-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)\end{pmatrix}, \tag{19}$$

$$-\frac{dlnY_x}{dln(1-t)} = -\frac{\beta^3(\gamma-1)}{(\gamma-\beta)^2}\frac{1}{\Delta^{TX}}, \tag{20}$$

where Δ^{TX} represents the determinant of the matrix in Eq. (18) as follows:

$$\Delta^{TX} = \frac{\beta^2(\gamma-1)}{(\gamma-\beta)^2}\left((\gamma-1)\left(1-\beta+\beta\left(2-\frac{\gamma-1}{\gamma-\beta}Y_e-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}(R_HY_e+R_H{}^*Y_e{}^*)\right) > 0.$$

From Eq. (12), we obtain the effects on the home agent's pre-transfer income as follows:

$$-\frac{dlnr_N}{dln(1-t)} = -\frac{\beta}{\gamma-\beta} - \frac{\gamma(1-\beta)}{\gamma-\beta}\frac{dlnQ}{dln(1-t)}, \tag{21}$$

$$-\frac{dlnr_H}{dln(1-t)} = -\frac{\beta}{\gamma-\beta} - \frac{\gamma(1-\beta)}{\gamma-\beta}\left(\frac{dlnQ}{dln(1-t)} + Y_e\frac{dlnY_x}{dln(1-t)}\right). \tag{22}$$

The first terms on the right-hand side of Eqs. (21) and (22) represent the direct negative effects of increasing the marginal tax rate. The second terms represent an indirect effect caused by changes in the market size.

Regarding the marginal effects of increasing income tax rates on Q, Y_x, r_N, and r_H from Eqs. (19)–(22), we obtain Proposition 1:[10]

Proposition 1. *Assuming that the home and foreign countries are completely symmetrical and have no policy* (t=t*=0), *a marginal increase in the income tax rates of the home government adopting the income tax scheme reduces the real revenues of all agents and the total real consumption* (*revenues*), *and raises the relative export volume of H-agents.*

Earlier studies refer to the reduction in aggregate income caused by the income redistribution system as "efficiency loss." An increase in the marginal tax rate first directly weakens the marginal revenues derived from working for all agents in the home country, and then their products and revenues. The domestic market shrinkage due to the decline in agents' revenues reduces their revenue further. However, the real revenue decline rate for H-agents is lower than for N-agents (i.e.,

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[10] The proofs of all the propositions are given in the appendix.

$-dlnr_H/dln(1-t)|_{t=t^*=0} > -dlnr_N/dln(1-t)|_{t=t^*=0}$) since H-agents alleviate the decline in their revenue by increasing their exports relative to the domestic supply. This implies that the income tax scheme enlarges the pre-transfer income gap between domestic agents (i.e., $-dln(r_H/r_N)/dln(1-t)|_{t=t^*=0}>0$).

From Eqs. (13), (21), and (22), we obtain the marginal effect of the income tax on the home agents' utility at $t=t^*=0$ as follows:

$$-\frac{du_N}{dln(1-t)}\bigg|_{t=t^*=0} = \left(\frac{R}{L} - r_N\right) - (1-\beta) r_N \frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0}, \tag{23}$$

$$-\frac{du_H}{dln(1-t)}\bigg|_{t=t^*=0} = \left(\frac{R}{L} - r_H\right) - (1-\beta) r_H \left(\frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0} + Y_e \frac{dlnY_x}{dln(1-t)}\bigg|_{t=t^*=0}\right). \tag{24}$$

The first terms on the right-hand side of Eqs. (23) and (24) represent the direct redistribution effect of the income tax. Since N-agents (H-agents) are net receivers (payers) in the tax-transfer system, the direct effect is always positive (negative) for them. The second term represents the effects caused by the change in their real pre-transfer revenues. H-agents' utility is always worsened by income tax since the direct and indirect effects are negative for them; however, N-agents' utility is improved when the direct positive effect overcomes the indirect negative effect. Finally, the marginal effects of income tax on total utilities in the home country, $U=(1-\chi)Lu_N+\chi Lu_H$, can be represented as follows:

$$\begin{aligned} -\frac{dU}{dln(1-t)}\bigg|_{t=t^*=0} &= R\left(\frac{\beta}{\gamma} - \left(1-\frac{\beta}{\gamma}\right)\left((1-R_H)\frac{dlnr_N}{dln(1-t)}\bigg|_{t=t^*=0} + R_H \frac{dlnr_H}{dln(1-t)}\bigg|_{t=t^*=0}\right)\right) \\ &= -(1-\beta)R\left(\frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0} + R_H Y_e \frac{dlnY_x}{dln(1-t)}\bigg|_{t=t^*=0}\right). \end{aligned} \tag{25}$$

From Eqs. (19)–(25), we obtain the following proposition.

Proposition 2. *Assuming that the home and foreign countries are completely symmetrical and have no policy* ($t=t^*=0$), *a marginal increase in the income tax rate of the home government adopting the income tax scheme* (i) *always worsens H-agents' utility*; (ii) *improves N-agents' utility when condition* (A) *is satisfied*; *and* (iii) *reduces total utilities in the home country. Condition* (A) *is presented by the following equation*:

$$2^{\frac{\gamma(1-\beta)}{\gamma-\beta}}\left(\frac{n_H}{n_N}\right)^{\frac{\beta\gamma}{\gamma-\beta}} > 1 + \frac{1}{\chi}. \qquad (26)$$

Proposition 2 (ii) shows that the income redistribution financed by the income tax improves N-agents' utility when the ability gap between N-agents and H-agents and the population share of H-agents are large enough to satisfy condition (A). This condition ensures that the size of income transfer from H-agents to N-agents is sufficiently large. Propositions 2 (i) and 2 (ii) imply that a redistribution system based on an income tax scheme reduces the utility inequality between domestic groups. We call this an equality improvement. Propositions 1 and 2 imply that the redistribution system improves equality in society at the cost of efficiency losses.

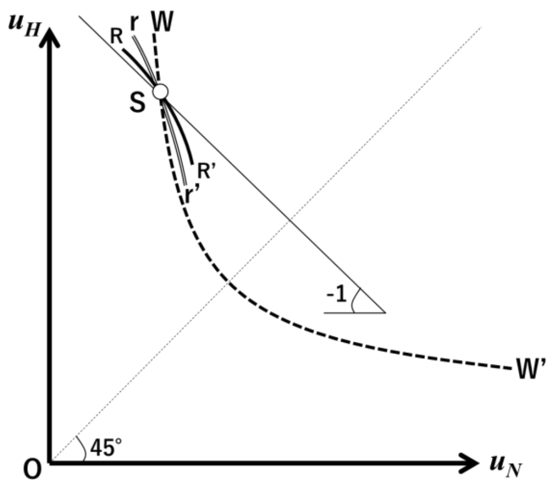

Fig. 1. Utility frontiers in the home country under the income tax scheme

The equity–efficiency trade-off with the income redistribution system is shown in Fig. 1. Point S in Fig. 1 represents the utility levels of N-agents and H-agents in the home country, in the absence of government policy. The curve RR' represents the utility frontier in the home country under the redistribution system based on the income tax scheme, assuming that χ=1/2. The curve RR' at point S is downward sloping since the income redistribution improves (worsens) N-(H-)agents' utility as shown in Propositions 2 (i) and 2 (ii).[11] The slope of the curve RR' at point S is below minus one since the income tax reduces the sum of all agents' utilities, as shown in Proposition 2 (iii).

When the government is inequality-averse, its social welfare function is defined as $W(u_N, u_H)$, where $W'(\cdot)>0$ and $W''(\cdot)<0$.[12] The social indifference curve derived from $W(u_N, u_H)$ has a convex shape in terms of origin, and the stronger the inequality-averse preference of the government, the stronger the convex shape of the curve. The social welfare effect of the redistribution system depends on the home government's aversion to inequality; when the government has strong inequality aversion preferences and its social indifference curve has a strong convex shape in terms of origin, such as the dotted line WW' in Fig. 1, the redistribution system improves social welfare. This indicates that the government values equality improvement over the efficiency losses caused by income tax. However, when the government has a strong efficiency preference, the shape of the social indifference curve is almost a straight line, and the redistribution system worsens social welfare.[13]

Last, we analyze the social welfare effects of the income tax scheme in the foreign country. From the comparative statics matrix (18), we obtain the marginal effects of an increase in income tax rates by the home government on the foreign aggregate real consumption (revenue) Q^* and the export volume to domestic supply ratio Y_x^* as follows:

$$-\frac{dlnQ^*}{dln(1-t)} = -\frac{\gamma\beta^2(1-\beta)}{(\gamma-\beta)^2} R_H{}^* Y_e{}^* \frac{1}{\Delta^{TX}} < 0, -\frac{dlnY_x{}^*}{dln(1-t)} = -\frac{\beta^3(\gamma-1)}{(\gamma-\beta)^2}\frac{1}{\Delta^{TX}} < 0. \quad (27)$$

[11] The utility frontier RR' arrives to the origin when the government levies all revenues from agents (i.e., t=1). Thus, the slope of the curve RR' changes from downward to upward if the income tax rates exceed the threshold level. This implies that N-agents' utility is improved by the redistributive tax policy only when the income tax rates are sufficiently low.
[12] The concavity of the social welfare function is justified by the following: In the case of two combinations of utilities with the same mean, society is expected to prefer low inequality between high and low utility agents (cf. Atkinson, 1970).
[13] Fig. 1 shows that if society does not care about equality, the government can improve social welfare by setting the income tax rates to negative. This policy is depicted as production subsidies financed by lump-sum tax. These subsidies offset agents' monopolistic distortion—a constant markup equal to $1/\beta$—and realize more efficient production (Itskhoki, 2008).

The shrinkage of the home country's market size due to income tax decreases foreign H-agents' export revenues. This leads to a reduction in the aggregate real revenues Q^* and relative exports by H-agents Y_x^*. The reduction in Q^* and Y_x^* decreases the real revenue and utilities of all agents in the foreign country. Therefore, we obtain the following proposition:

Proposition 3. *When the home government implements income redistribution under the income tax scheme, foreign social welfare deteriorates*.

3.3. The case where the home and foreign governments both implement the income tax scheme

Propositions 2 and 3 imply that the income tax scheme in the home country improves its social welfare but worsens that of the foreign country. Therefore, when the foreign government implements the same scheme as the home government, both countries may worsen their social welfare by transferring welfare losses to each other.

From Eqs. (18) and (21)–(25), when the foreign government raises income tax rates, similar to that of the home government, the marginal effects of an increase in income tax rates by the home government on Q, Y_x, r_N, r_H, u_N, u_H, and U are contained in the following proposition.

Proposition 4. *Assuming that the home and foreign countries are completely symmetrical and have no policy* (t=t^*=0), *a marginal increase in the income tax rates of both countries adopting the income tax scheme* (i) *reduces the real revenues of all agents and total real consumption* (*revenues*), *whereas the relative export volume by H-agents is unchanged*; (ii) *worsens H-agents' utility and improves N-agents' utility if condition* (A) *is satisfied*; *and* (iii) *reduces total utilities in both countries*.

Proposition 4 implies that the income tax scheme improves equality at the cost of efficiency, even when the foreign government implements the same scheme. This means that the slope of the utility frontier curve when both countries implement the income tax scheme at point S in Fig. 1 is negative. Now, we define an “equality improvement cost” by the size of the reduction in H-agents’ utility required to improve N-agents’ utility by one unit (i.e., the scale of the slope of the utility frontier curve ($-du_H/du_N$) at point S in Fig. 1). By comparing the equality improvement costs accompanied by the income tax scheme in the case where only the home government implements that scheme with that where both governments implement the scheme, we obtain the following proposition.

Proposition 5. *Assuming that the home and foreign countries are completely symmetrical and that condition* (A) *is satisfied, the equality improvement cost accompanied by the income tax scheme in the home country is larger when the home and foreign governments both implement the same income tax scheme than when only the home government implements the scheme.*

The double curve rr’ in Fig. 1 is the utility frontier in the home country when both governments implement the same income tax scheme. Proposition 5 implies that the slope of the curve rr’ at point S is steeper than that of the curve RR’. This indicates that the social welfare effect of the income redistribution system under the income tax scheme is smaller when both governments implement the same scheme to improve domestic social welfare than when only the home government implements that scheme. This is because home and foreign governments both inflict social welfare losses. However, when both countries have sufficiently strong inequality-averse preferences, they can improve social welfare by cooperatively raising income tax rates.

4. The import tariff scheme

In this section, we analyze the income redistribution system based on the import tariff scheme and compare its social welfare effects with those of the income tax scheme.

4.1. Trade equilibrium and comparative statics system

When governments use import tariffs instead of income tax (i.e., t=t*=0), from Eqs. (5), (8), and (11), we derive the following trade balance condition:

$$(1+T){Y_x}^{1-\beta} = \frac{\chi^* L^*}{\chi L}\left(\frac{{Y_x}^*}{Y_x}\right)^{\beta}\left(\frac{1+Y_x}{1+{Y_x}^*}\right)^{\frac{\beta(\gamma-1)}{\gamma-\beta}}\left(\frac{Q^*}{Q}\right)^{\frac{\beta(1-\beta)}{\gamma-\beta}}\left(\frac{{n_H}^*}{n_N}\right)^{\frac{\beta\gamma}{\gamma-\beta}}. \tag{28}$$

We obtain the real tariff revenue received by the home government TA as follows:

$$TA = \frac{TL^*\chi^* {p_{He}}^* {y_{He}}^*}{P} = \frac{TL\chi p_{He} y_{He}}{P} = TL\chi {y_{He}}^{\beta}{Y_x}^{1-\beta}Q^{1-\beta} = TL\chi\frac{Y_x}{1+Y_x}r_H. \tag{29}$$

where the second equality sign uses the trade balance condition, and the third equality sign uses the definitions of Y_x* and Eq. (9).

From Eqs. (12) and (29), the home country's aggregate real revenue is

$$R = (1-\chi)Lr_N + \chi Lr_H + TA = (1-\chi)Lr_N + \chi L\left(1+\frac{TY_x}{1+Y_x}\right)r_H$$

$$= L\beta^{\frac{\beta}{\gamma-\beta}}Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}}\left((1-\chi){n_N}^{\frac{\beta\gamma}{\gamma-\beta}} + \chi\left(1+\frac{TY_x}{1+Y_x}\right)(1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}}{n_H}^{\frac{\beta\gamma}{\gamma-\beta}}\right). \tag{30}$$

The same applies to the foreign country. Thus, we obtain the following equations to show that aggregate real revenue equals aggregate real consumption:

$$Q^{\frac{\beta(\gamma-1)}{\gamma-\beta}} = \beta^{\frac{\beta}{\gamma-\beta}}\left((1-\chi){n_N}^{\frac{\beta\gamma}{\gamma-\beta}} + \chi\left(1+\frac{TY_x}{1+Y_x}\right)(1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}}{n_H}^{\frac{\beta\gamma}{\gamma-\beta}}\right)L, \tag{31}$$

$$Q^{*\frac{\beta(\gamma-1)}{\gamma-\beta}} = \beta^{\frac{\beta}{\gamma-\beta}}\left((1-\chi^*){n_N}^{*\frac{\beta\gamma}{\gamma-\beta}} + \chi^*\left(1+\frac{T^*{Y_x}^*}{1+{Y_x}^*}\right)(1+{Y_x}^*)^{\frac{\gamma(1-\beta)}{\gamma-\beta}}{n_H}^{*\frac{\beta\gamma}{\gamma-\beta}}\right)L^*. \tag{32}$$

We obtain the equilibrium values Q, Q*, Y_x, and Y_x* under the import tariff scheme from Eqs. (9), (28), (31), and (32). From these equations, we obtain the following comparative statics matrix:

$$\begin{bmatrix} a_{11} & -a_{11} & a_{13} & a_{14} \\ a_{21} & 0 & {a_{23}}^{TR} & 0 \\ 0 & a_{21} & 0 & {a_{34}}^{TR} \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} dlnQ \\ dlnQ^* \\ dlnY_x \\ dln{Y_x}^* \end{bmatrix} = \begin{bmatrix} {b_1}^{TR} \\ {b_2}^{TR} \\ {b_3}^{TR} \\ {b_4}^{TR} \end{bmatrix}, \tag{33}$$

where ${a_{23}}^{TR} = -R_{HT}C_e\left(1-\beta+\beta\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)\right)$, ${a_{34}}^{TR} = -{R_{HT}}^*{C_e}^*\left(1-\beta+\beta\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{{Y_e}^*}{{C_e}^*}\right)\right)$,

${b_1}^{TR} = -dln(1+T)$, ${b_2}^{TR} = R_{HT}C_e dln(1+T)$, ${b_3}^{TR} = {R_{HT}}^*{C_e}^* dln(1+T^*)$,

$$b_4{}^{TR} = -\frac{1}{1-\beta}\left(dln(1+T) + dln(1+T^*)\right),\ C_e = \frac{(1+T)Y_e}{1+TY_e},\ C_e{}^* = \frac{(1+T^*)Y_e{}^*}{1+T^*Y_e{}^*},$$

$$R_{HT} = \frac{\chi L r_H + TA}{R},\ R_{HT}{}^* = \frac{\chi^* L^* r_H{}^* + TA^*}{R^*}.$$

R_{HT} represents the share of the sum of the aggregate tariff revenue and H-agents' real revenue relative to the aggregate real revenue (R_{HT}=R_H at T=T^*=0).

4.2. The case where only the home government implements the import tariff scheme

When the foreign government has a free trade policy (i.e., T^*=0), from the comparative statics matrix (33), we obtain the elasticity of the aggregate real consumption (revenue) Q and the export volume to domestic supply ratio Y_x in the home country with respect to the import tariff rate T as follows:

$$\frac{dlnQ}{dln(1+T)} = \frac{\beta^2}{\gamma-\beta}\frac{R_{HT}C_e}{\Delta^{TR}}\left(\begin{array}{c}(\gamma-1)\left(\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e\right)-\left(1+\frac{\beta}{1-\beta}\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)\right)\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)\right)\\ -\frac{\gamma(1-\beta)}{\gamma-\beta}R_{HT}{}^*Y_e{}^*\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)\end{array}\right), \quad (34)$$

$$\frac{dlnY_x}{dln(1+T)} = -\frac{\beta^2(\gamma-1)}{(\gamma-\beta)^2}\frac{1}{\Delta^{TR}}\left(\gamma-1+\frac{\beta(\gamma-1)}{1-\beta}\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)+\frac{\gamma(1-\beta)}{\gamma-\beta}R_{HT}{}^*Y_e{}^*+(1-\beta)R_{HT}C_e\right), \quad (35)$$

$$\Delta^{TR} = \frac{\beta^2(\gamma-1)}{(\gamma-\beta)^2}\left(\begin{array}{c}(\gamma-1)\left(1-\beta+\beta\left(2-\frac{\gamma-1}{\gamma-\beta}Y_e-\frac{\gamma-1}{\gamma-\beta}Y_e{}^*\right)\right)\\ +(1-\beta)^2R_{HT}C_e\left(1+\frac{\beta}{1-\beta}\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_{HT}{}^*Y_e{}^*\end{array}\right)>0.$$

where Δ^{TR} represents the matrix determinant in Eq. (33).[14]

When the home and foreign countries are completely symmetrical, from Eqs. (11), (12), (34), and (35), we obtain the proposition for the marginal effects of the import tariff rates on Q, Y_x, r_H, r_N, y_N, y_H, y_{Hd}, and y_{He} as follows.

[14] Since Y_e=C_e and $Y_e{}^*$=$C_e{}^*$ at T=T^*=0, we know that Δ^{TX}=Δ^{TR} at t=t^*=T=T^*=0.

Proposition 6. *Assuming that the home and foreign countries are completely symmetrical and have a free trade policy* (T=T*=0), *a marginal increase in import tariff rates by the home government adopting the import tariff scheme* (i) *increases production outputs and real revenues of N-agents*; (ii) *reduces export volumes, total product outputs, and real revenues of H-agents, although their product outputs for domestic market increase*; *and* (iii) *reduces the relative export volume by H-agents and increases total real consumption* (*revenues*) *in the home country*.

Proposition 6 (iii) shows that, unlike the income tax scheme, the import tariff scheme produces efficiency "gains" for the home country. This is the same as in past studies on the two-country trade model. Propositions 6 (i) and (ii) represent the distributional effects of import tariffs. An increase in import product prices due to the import tariff gives domestic agents an incentive to expand their outputs for the domestic market, whereas exports by H-agents are reduced.[15] Since the latter effect surpasses the former effect, H-agents' real revenues decrease.[16] The real revenues of N-agents increase because their products are supplied only to the domestic market. The import tariff scheme therefore reduces the pre-transfer income gap between N-agents and H-agents, unlike the income tax scheme. Real tariff revenues increase when tariff rates are sufficiently low. Since the sum of the increases in N-agents' real revenues and real tariff revenues is above the decreases in H-agents' real revenues, total real revenues in the home country increase.

From Eqs. (13) and (29), the marginal effects of the import tariff on the home agents' utilities at T=T*=0 are as follows:

$$\left.\frac{du_N}{dln(1+T)}\right|_{T=T^*=0}=\frac{R}{L}\left(R_{HT}Y_e+\left(1-\frac{\beta}{\gamma}\right)\frac{1-R_{HT}}{1-\chi}\left.\frac{dlnr_N}{dln(1+T)}\right|_{T=T^*=0}\right), \tag{36}$$

$$\left.\frac{du_H}{dln(1+T)}\right|_{T=T^*=0}=\frac{R}{L}\left(R_{HT}Y_e+\left(1-\frac{\beta}{\gamma}\right)\frac{R_{HT}}{\chi}\left.\frac{dlnr_H}{dln(1+T)}\right|_{T=T^*=0}\right). \tag{37}$$

[15] Tariffs reduce imports from the foreign country, but this is accompanied by a decrease in exports from the home country due to trade balance conditions.
[16] Since $Y_e \to 0$ and $C_e \to 1$ when $T\to\infty$, we know from Eq. (34) that $dlnQ/dlnT|_{T\to\infty}=-\beta^3(\gamma-1)R_H/\{(\gamma-\beta)(1-\beta)\Delta^{IT}\}<0$. This implies that total real consumption is increased by the import tariff policy only when the import tariff rates are below the threshold rates.

The first terms in parentheses on the right-hand side of Eqs. (36) and (37) represent the utility improvements derived from the transfer income received from the government financed by tariff revenues. The second term represents the effects of changes in real revenue. The marginal effect of the import tariff on the total utilities of the home agents at T=T*=0 is as follows:

$$\left.\frac{dU}{dln(1+T)}\right|_{T=T^*=0} = R\left(R_{HT}Y_e + \left(1-\frac{\beta}{\gamma}\right)\left((1-R_{HT})\left.\frac{dlnr_N}{dln(1+T)}\right|_{T=T^*=0} + R_{HT}\left.\frac{dlnr_H}{dln(1+T)}\right|_{T=T^*=0}\right)\right)$$

$$= R\left(R_{HT}Y_e\left(1+(1-\beta)\left.\frac{dlnY_x}{dln(1+T)}\right|_{T=T^*=0}\right) + (1-\beta)\left.\frac{dlnQ}{dln(1+T)}\right|_{T=T^*=0}\right). \qquad (38)$$

From Eqs. (34)–(38), we obtain the following proposition.

Proposition 7. *Assuming that the home and foreign countries are completely symmetrical and have a free trade policy* (T=T*=0), *a marginal increase in import tariff rates by the home government adopting the import tariff scheme* (i) *always improves N-agents' utility*; (ii) *worsens H-agents' utility unless the population share of H-agents is large* (*i.e.,* $du_H/dln(1+T)|_{T=T^*=0}<0$ *if* $\chi<\chi^+$, *where* $1/2<\chi^+<1$); *and* (iii) *increases the total utilities of home agents*.

Propositions 7 (i) and (ii) show that, as in the case of the income tax scheme, the import tariff scheme also results in an improvement in equality between domestic groups. The utility of N-agents is improved by increasing the import tariff rate, because their pre-transfer real revenue is also increased and they receive the transfer income from their government. H-agents' utility will decline unless they receive a sufficiently large transfer income from the government since the import tariff has decreased their pre-transfer real revenue. Therefore, unless the population ratio of the exportable H-agents is large and the trade volume is extremely large such that the home government obtains sufficiently large tariff revenues from imports, H-agents' utility is not improved by the import tariff scheme. However, if all agents are exportable, their utility is improved by import tariffs. Proposition 7 (iii) shows that, even if H-agents' utility worsens, the sum of increases in N-agents' utility always exceeds the sum of the decreases in H-agents' utility because the import tariff increases the home country's total revenue.

Propositions 6 and 7 show that the income redistribution system based on the import tariff scheme resolves the equality–efficiency trade-off, that is, it can realize not only an equality improvement but also induce efficiency gains. We compare the social welfare effects of the income tax scheme with those of the import tariff scheme in Fig. 2. Curve TT' represents the utility frontier in the home country with the import tariff scheme. The slope of this curve at point S is downward because the import tariff improves the utility of N-agents but worsens that of H-agents.[17] However, the scale of its slope is greater than minus one because total utility is increased by the import tariff. Social welfare in the home country is improved by the import tariff scheme, regardless of the government's inequality aversion preference (Fig. 2).

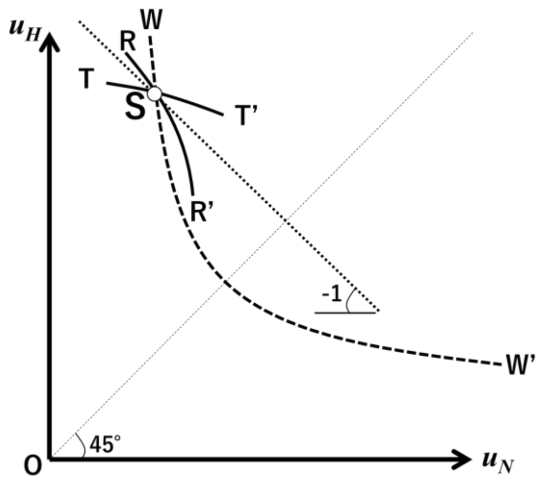

Fig. 2. Utility frontiers in the home country under the income tax and import tariff schemes

[17] The slope of the utility frontier TT' changes from downward to upward if the import tariff rates exceed the threshold level. This is because utility levels of N-agents and H-agents when the home country is a closed economy (i.e., t=∞) is lower than when the home government has a free trade policy.

The equality improvement cost accompanied by the income redistribution system is lower under the import tariff scheme than under the income tax scheme when χ=1/2 (Fig. 2). Comparing equality improvement costs of the income tax and import tariff schemes, we obtain the following proposition.

Proposition 8. *Assuming that the home and foreign countries are completely symmetrical and that condition* (A) *and* $\chi<\chi^{+}$ *are satisfied, the equality improvement cost of income redistribution under the import tariff scheme is smaller than that under the income tax scheme*.

Proposition 8 implies that the import tariff scheme is a more efficient method for income redistribution than the income tax scheme. Therefore, as Fig. 2 shows, the income redistribution under the import tariff scheme can realize higher social welfare than the income tax scheme. There are two reasons for this. First, the scale of the income transfer required to reduce the utility gap between N-agents and H-agents is smaller in the import tariff scheme than in the income tax scheme, since the pre-transfer income gap between N-agents and H-agents is reduced under the import tariff scheme, and expanded under the income tax scheme. Second, the efficiency gains caused by import tariffs compensate for a portion of H-agents' income losses caused by the income redistribution.

As for the welfare effects of the import tariff on the foreign country, from the comparative statics matrix (33), we find that the increase in import tariff rates in the home country reduces foreign aggregate real consumption (revenue) and the export volume relative to domestic supply as follows:

$$\frac{dlnQ^{*}}{dln(1+T)} = -\frac{\beta^{2}\gamma(1-\beta)}{(\gamma-\beta)^{2}}\frac{{R_{HT}}^{*}{Y_e}^{*}}{\Delta^{TR}}\left(R_{HT}C_e\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)+\frac{\gamma-1}{1-\beta}\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e\right)\right)<0, \tag{39}$$

$$\frac{dln{Y_x}^{*}}{dln(1+T)} = -\frac{\beta^{3}(\gamma-1)}{(\gamma-\beta)^{2}}\frac{1}{\Delta^{TR}}\left(R_{HT}C_e\left(1-\frac{\gamma-1}{\gamma-\beta}\frac{Y_e}{C_e}\right)+\frac{\gamma-1}{1-\beta}\left(1-\frac{\gamma-1}{\gamma-\beta}Y_e\right)\right)<0. \tag{40}$$

The reduction in the export revenue of foreign agents due to the import tariff in the home government reduces the aggregate real revenue in the foreign country. As in the case of the income tax scheme, we obtain the following proposition.

Proposition 9. *When the home government implements income redistribution under an import tariff scheme, foreign social welfare declines*.

Proposition 9 shows that the import tariff scheme is a "beggar-thy-neighbor" policy; the home government improves its social welfare at the expense of foreign social welfare. This coincides with the literature on the standard two-country trade model. However, we need to establish whether the income tax scheme is also a "beggar-thy-neighbor" policy, since it also improves home social welfare at the expense of foreign social welfare.

4.3. The case where both the home and foreign governments implement the import tariff scheme

From Eqs. (33), (36), and (37), the marginal effects of an increase in import tariff rates by the home government on Q, Y_x, r_N, r_H, u_N, u_H, and U when the foreign government raises import tariff rates, similar to that of the home government, are obtained as follows.

Proposition 10. *Assuming that the home and foreign countries are completely symmetrical and have a free trade policy* (T=T*=0), *a marginal increase in the import tariff rates of both countries* (i) *reduces real revenues of all agents, total real consumption* (*revenues*)*, and relative export volume of H-agents*; (ii) *worsens H-agents' utility and improves N-agents' utility*; *and* (iii) *reduces total utilities of the agents*.

Proposition 10 shows that the import tariff scheme induces "efficiency losses" when the foreign government also raises import tariffs—by raising import tariffs the governments cause economic losses to each other. This implies that the mutual raising of import tariffs results in welfare losses for both countries when society regards efficiency as the index of welfare, similar to the standard two-country trade model. The efficiency losses cause a reduction in the pre-transfer real revenues of not only H-agents but also N-agents. However, the utility of N-agents is improved even when the foreign government also raises import tariffs, since their transfer income received from their government is larger than their revenue losses. Therefore, import tariffs induce an improvement in equality. From this, when the home and foreign governments have sufficiently strong inequality-averse preferences, they can improve social welfare by cooperatively raising import tariffs even when their total utilities are reduced, similar to the income tax scheme.

Proposition 10 implies that the slope of the utility frontier in the home country when both governments implement the same import tariff scheme is below minus one at point S in Fig. 2. This means that the equality improvement cost is larger when both governments implement the import tariff scheme than when only the home government implements the scheme. Comparing the equality improvement costs of the income tax scheme and the import tariff scheme when both governments implement the same scheme, we obtain the following proposition.

Proposition 11. *Assuming that the home and foreign countries are completely symmetrical and condition* (A) *is satisfied, when both governments implement the same scheme, the equality improvement cost of the income redistribution system under the import tariff scheme is smaller than that under the income tax scheme.*

Proposition 11 implies that even when both governments implement the same scheme, the import tariff scheme is a more efficient method for income redistribution and can realize higher social welfare than the income tax scheme. This is because the scale of income transfer required to reduce the utility gap between N-agents and H-agents is smaller under the import tariff scheme than under the income tax scheme, since import tariffs decrease pre-transfer income gaps, while income taxes do not.

5. The income tax scheme levied only on the high-income group

In an open economy, agents are clearly separated into exporters and non-exporters; therefore, the government can easily levy different income tax rates on exporters and non-exporters. In this section, we analyze the income tax scheme levied only on H-agents who export their variety and earn high revenues. We assume that the home (foreign) government levies a marginal income tax rate t_H (t_H *) on only H-agents and transfers the tax revenue evenly to all domestic agents. The pre-transfer real revenue and utility of agents are represented by

$$r_N = \beta^{\frac{\beta}{\gamma-\beta}} Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}} n_N^{\frac{\beta\gamma}{\gamma-\beta}},\ r_H = \beta^{\frac{\beta}{\gamma-\beta}} (1-t_H)^{\frac{\beta}{\gamma-\beta}} Q^{\frac{\gamma(1-\beta)}{\gamma-\beta}} (1+Y_x)^{\frac{\gamma(1-\beta)}{\gamma-\beta}} n_H^{\frac{\beta\gamma}{\gamma-\beta}}, \tag{41}$$

$$u_N = \left(1-\frac{\beta}{\gamma}\right) r_N + t_H \chi r_H,\ u_N = \left(1-\frac{\beta}{\gamma}\right)(1-t_H) r_H + t_H \chi r_H. \tag{42}$$

The comparative statics matrix is obtained as follows:

$$\begin{bmatrix} a_{11} & -a_{11} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23}{}^{TX} & 0 \\ 0 & a_{21} & 0 & a_{34}{}^{TX} \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} dlnQ \\ dlnQ^* \\ dlnY_x \\ dlnY_x{}^* \end{bmatrix} = \begin{bmatrix} b_1{}^{TH} \\ b_2{}^{TH} \\ b_3{}^{TH} \\ 0 \end{bmatrix}, \tag{43}$$

where $b_1{}^{TH} = \frac{\beta}{\gamma-\beta}\left(dln(1-t_H{}^*) - dln(1-t_H)\right)$, $b_2{}^{TH} = \frac{\beta}{\gamma-\beta} R_H dln(1-t_H)$, and

$b_3{}^{TH} = \frac{\beta}{\gamma-\beta} R_H{}^* dln(1-t_H{}^*)$.

From Eq. (43), similar to Proposition 2, the utility gap between N-agents and H-agents in the home country decreases when only the home government raises its marginal income tax rates t_H (i.e., $-du_N/dln(1-t_H)>0$, $-du_H/dln(1-t_H)<0$ at $t_H=t_H{}^*=0$). Comparing the equality improvement cost in the case where the home government raises the marginal tax rates only on exporter t_H, the uniform marginal tax rates t, and the import tariff rates T, we obtain the following proposition.

Proposition 12. *Assuming that the home and foreign countries are completely symmetrical and condition* (A) *is satisfied, when only the home government implements income redistribution, the* *equity improvement cost of the income tax on exporters is smaller than income tax on all agents but larger than for import tariffs.*

Proposition 12 implies that an income tax on exporters only is a more efficient way to reduce inequality between high- and low-income groups than a uniform income tax on all agents. This is because the income tax on exporters decreases the pre-transfer income gap between N-agents and H-agents, since N-agents have no tax burden and their incentives to earn income are not weakened by taxation. However, since the income tax on exporters also has efficiency losses, its equality improvement cost is larger than for import tariffs.

Last, by comparing the equality improvement costs when both governments raise marginal tax rates on exporters only and import tariff rates, we obtain the following proposition.

Proposition 13. *Assuming that the home and foreign countries are completely symmetrical and condition* (A) *is satisfied, when both countries implement the same income redistribution system, the equality improvement cost of an income tax on exporters is equal to that of import tariffs.*

Proposition 13 shows that the difference in effectiveness as a financial resource of the income redistribution between an income tax on exporters and an import tariff is virtually eliminated when the foreign government implements the same scheme. In our model, as the Lerner symmetry theorem suggests (Lerner, 1936), an equivalence between import tariffs and export taxes is established.[18] When both governments implement the same income redistribution system, the export tax and the income tax imposed on exporters can essentially be regarded as the same tax-transfer system because the equality improvement cost is equal between the two tax-transfer schemes. The difference between the income tax on exporters and the export tax lies in its effect on efficiency when only one government implements the tax-transfer scheme. The income tax on exporters induces efficiency

[18] Specifically, assuming home and foreign governments impose ad valorem export tax rates T_x(=T/(1+T)) and T_x*(=T*/(1+T*)), we obtain the same results and propositions as in the case of the import tariff scheme.

losses, while the export tax (import tariff) creates efficiency gains by depriving economic rents from the trade partner country through the improvement of the terms of trade as represented by the standard trade model. The efficiency gains reduce the equality improvement cost of the export tax (import tariff) compared to the income tax on exporters.

6. Conclusions

This study compares the social welfare effects of income redistribution systems financed by income taxes and import tariffs. Both tax-transfer schemes improve domestic equality by increasing the utility of low-income agents at the cost of high-income agents' utility by distributing taxes levied evenly in proportion to income or import consumption. However, we show that the import tariff scheme improves domestic inequality more efficiently than the income tax scheme, regardless of whether one or both governments implement the scheme. In the case where only one country implements an income redistribution system, although the income tax scheme faces an equality–efficiently trade-off, the import tariff scheme not only improves domestic equality but also increases domestic efficiency. This equality–efficiency trade-off under the import tariff scheme is lost when the trade partner country implements the same income redistribution system. However, even in the latter case, we show that efficiency losses caused by taxation are smaller under the import tariff scheme than under the income tax scheme.

We find that the import tariff scheme has two advantages over the income tax scheme. First, the import tariff scheme reduces pre-transfer incomes between high- and low-income groups because, as the Lerner symmetry theorem suggests (Lerner, 1936), the import tariff has the same distributional effects as the export tax and has properties similar to a progressive tax system that imposes taxes on the tradeable high-income group. Second, if other countries do not react, the import tariff scheme resolves the equality–efficiency trade-off accompanied by the income tax scheme by increasing aggregate real income. According to the standard trade theory, an import tariff scheme is regarded as an undesirable policy even if it creates economic gains for a country since it induces economic losses when the other country also raises import tariffs. However, we need to highlight the following two points. First, when equality is included as a factor of social welfare and import tariffs are used as financial resources for the income redistribution system, import tariffs can improve social welfare even if other countries similarly raise the import tariff. Second, the income redistribution system based on income tax also has the same problem as that based on import tariff in that it causes economic losses to other countries. Therefore, we need to explore how countries design policy coordination regardless of whether income tax or import tariff is used, rather than whether we should introduce a protective import tariff policy or a free trade policy with the income redistribution system. To explore optimal international policy coordination, we need to introduce a social welfare function to evaluate policies. This is a challenge for further research.

Appendix.

Proof of Proposition 1. Since $(\gamma\text{-}1)/(\gamma\text{-}\beta)<1$ (from $\gamma>1$ and $\beta<1$) and Y_e, $Y_e^*<1$, we have $\Delta^{TX}>0$. Thus, we can easily show that $-dlnY_x/dln(1\text{-}t)<0$ from Eq. (20).

When $L=L^*$, $\chi=\chi^*$, $n_H=n_H^*$, and $t=t^*=0$, we obtain Q=Q^* and $Y_x=Y_x^*=1$ from Eqs. (9) and (14)–(16). Since $R_H=R_H^*<1$ and $Y_e=Y_e^*=1/2$, from Eq. (19), we have $-dlnQ/dln(1\text{-}t)$ at $t=t^*=0$ as follows:

$$-\frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0}=-\frac{\beta^2}{(\gamma-\beta)^2}\frac{1}{2\Delta^{TX}}\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(2-\frac{\gamma(1-\beta)}{\gamma-\beta}R_H+2\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)<0. \quad \text{(A.1)}$$

From Eqs. (21) and (A.1), we have $-dlnr_N/dln(1\text{-}t)<0$ at $t=t^*=0$. From Eqs. (20), (22), and (A.1), we obtain $-dlnr_H/dln(1\text{-}t)<0$ at $t=t^*=0$ since

$$-\frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0}-Y_e\frac{dlnY_x}{dln(1-t)}\bigg|_{t=t^*=0}=-\frac{\beta^2}{(\gamma-\beta)^2}\frac{1}{2\Delta^{TX}}\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(1-\frac{\gamma(1-\beta)}{\gamma-\beta}R_H+2\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)<0. \quad \text{(A.2)}$$

Q.E.D.

Proof of Proposition 2. From Eqs. (24) and (A.2), we have $-du_H/dln(1-t)<0$ at $t=t^*=0$. From Eqs. (23) and (A.1), we obtain $-du_N/dln(1-t)$ at $t=t^*=0$ as follows:

$$-\frac{du_N}{dln(1-t)}\bigg|_{t=t^*=0}=\frac{R}{L}-\left(1-\frac{\beta}{\gamma}\right)\left(1+\frac{dlnr_N}{dln(1-t)}\right)r_N=\frac{R}{L}\left(1-\left(1+(1-\beta)\frac{dlnQ}{dln(1-t)}\right)\frac{1-R_H}{1-\chi}\right)$$

$$=\frac{\beta^2}{(\gamma-\beta)^2}\frac{R}{L}\frac{1}{2\Delta^{TX}}\left(\left(1-\frac{1-R_H}{1-\chi}\right)J-\frac{1-R_H}{1-\chi}K\right), \tag{A.3}$$

where $J=2(\gamma-1)\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)>0$ and

$$K=(1-\beta)\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(2-\frac{\gamma(1-\beta)}{\gamma-\beta}R_H+2\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)>0.$$

From Eq. (A.3), we obtain $-du_N/dln(1-t)|_{t=t^*=0}>0$ when $1-(1-R_H)/(1-\chi)>(1-R_H)/(1-\chi)$ and $J>K$. From $R_H=\chi Lr_H/R$, we obtain $1-(1-R_H)/(1-\chi)>(1-R_H)/(1-\chi)$ when $r_H/r_N>(1+1/\chi)$. From Eq. (12) and $Y_x=1$, we find the condition for $1-(1-R_H)/(1-\chi)>(1-R_H)/(1-\chi)$ as follows:

$$2^{\frac{\gamma(1-\beta)}{\gamma-\beta}}\left(\frac{n_H}{n_N}\right)^{\frac{\beta\gamma}{\gamma-\beta}}>1+\frac{1}{\chi}. \tag{A.4}$$

We obtain the differences between J and K as follows:

$$J-K=(\gamma-2+\beta)\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+2(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)+2\frac{\gamma(\gamma-1)(1-\beta)^2}{\gamma-\beta}R_H. \tag{A.5}$$

From Eq. (A.5), we find that $J>K$, because $\gamma>2$.

Finally, from Eqs. (25) and (A.2), we know that $-dU/dln(1-t)|_{t=t^*=0}<0$ Q.E.D.

Proof of Proposition 4. From Eq. (18), we obtain $-dlnQ/dln(1-t)$ and $-dlnY_x/dln(1-t)$ at $t=t^*=0$ when $dln(1-t)=dln(1-t^*)$ as follows:

$$-\frac{dlnQ}{dln(1-t)}\bigg|_{\substack{t=t^*=0\\ dln(1-t)=dln(1-t^*)}}=-\frac{1}{\gamma-1}<0 \text{ and } -\frac{dlnY_x}{dln(1-t)}\bigg|_{\substack{t=t^*=0\\ dln(1-t)=dln(1-t^*)}}=0. \tag{A.6}$$

From Eqs. (21), (22), and (A.6), we know that $-dlnr_N/dln(1-t)=-dlnr_H/dln(1-t)<0$ at $t=t^*=0$ when $dln(1-t)=dln(1-t^*)$. From the above and Eqs. (24) and (25), it is easily found that $-du_H/dln(1-t)$, $-dU/dln(1-t)<0$ at $t=t^*=0$, and $dln(1-t)=dln(1-t^*)$. For $-du_N/dln(1-t)$, from Eqs. (23) and (A.6), we obtain

$$-\frac{du_N}{dln(1-t)}\bigg|_{\substack{t=t^*=0\\ dln(1-t)=dln(1-t^*)}} = \frac{R}{L}\left(1-\frac{1-R_H}{1-\chi}-\frac{1-\beta}{\gamma-1}\frac{1-R_H}{1-\chi}\right). \tag{A.7}$$

Since $(1-\beta)/(\gamma-1)<1$ ($\because$ $\gamma>2$) and $1-(1-R_H)/(1-\chi)>(1-R_H)/(1-\chi)$, when condition (A) is satisfied, we know that $-du_N/dln(1-t)>0$ at $t=t^*=0$ and $dln(1-t)=dln(1-t^*)$. Q.E.D.

Proof of Proposition 5. From Eqs. (A.1) and (A.6), we obtain

$$-\frac{dlnQ}{dln(1-t)}\bigg|_{\substack{t=t^*=0\\ dln(1-t)=dln(1-t^*)}} - \left(-\frac{dlnQ}{dln(1-t)}\bigg|_{t=t^*=0}\right) = -\frac{R_H}{2\Delta^{TX}}\frac{\beta^2\gamma(1-\beta)}{(\gamma-\beta)^2} < 0. \tag{A.8}$$

From Eqs. (20) and (A.6), we know that $-dlnY_x/dln(1-t)|_{t=t^*=0,\ dln(1-t)=dln(1-t^*)}<-dlnY_x/dln(1-t)|_{t=t^*=0}$. From the above and Eqs. (23) and (24), we find that $-du_i/dln(1-t)|_{t=t^*=0,\ dln(1-t)=dln(1-t^*)}<-du_i/dln(1-t)|_{t=t^*=0}$. In addition, from $-du_N/dln(1-t)|_{t=t^*=0,\ dln(1-t)=dln(1-t^*)}$, $-du_N/dln(1-t)|_{t=t^*=0}>0$, $-du_H/dln(1-t)|_{t=t^*=0,\ dln(1-t)=dln(1-t^*)}$, and $-du_H/dln(1-t)|_{t=t^*=0}<0$, we know that $-(du_H/du_N|_{t=t^*=0,\ dln(1-t)=dln(1-t^*)})>-(du_H/du_N|_{t=t^*=0})$. Q.E.D.

Proof of Proposition 6. We know that $dlnY_x/dln(1+T)<0$ from Eq. (35). Since $Y_e/C_e=1$ and $R_{HT}=R_H$ when $T=0$, from Eq. (34) we find that $dlnQ/dlnT$ at $T=T^*=0$ when the home and foreign countries are symmetrical, as follows:

$$\frac{dlnQ}{dln(1+T)}\bigg|_{T=T^*=0} = \frac{\beta^2\gamma(1-\beta)^2}{(\gamma-\beta)^3}\frac{R_H}{4\Delta^{TR}}\left(\frac{(\gamma-1)\left((\gamma-\beta)^2-(\gamma-\beta^2)\right)}{\gamma(1-\beta)^2}-R_H\right). \tag{A.9}$$

Since $R_H<1$ and $(\gamma-1)((\gamma-\beta)^2-(\gamma-\beta^2))/(\gamma(1-\beta)^2)>1$ when $\gamma>2$, we find that $dlnQ/dln(1+T)|_{T=T^*=0}>0$.

From Eqs. (11) and (12) and the definition of Y_x, we obtain the effects of the import tariff rates on product outputs and real revenues of agents in the home country as follows:

$$\frac{dlny_N}{dln(1+T)} = \frac{1-\beta}{\gamma-\beta}\frac{dlnQ}{dln(1+T)},\ \frac{dlny_H}{dln(1+T)} = \frac{1-\beta}{\gamma-\beta}\left(\frac{dlnQ}{dln(1+T)}+Y_e\frac{dlnY_x}{dln(1+T)}\right), \tag{A.10}$$

$$\frac{dlnr_N}{dln(1+T)} = \frac{\gamma(1-\beta)}{\gamma-\beta}\frac{dlnQ}{dln(1+T)},\ \frac{dlny_H}{dln(1+T)} = \frac{\gamma(1-\beta)}{\gamma-\beta}\left(\frac{dlnQ}{dln(1+T)}+Y_e\frac{dlnY_x}{dln(1+T)}\right), \tag{A.11}$$

$$\frac{dlny_{He}}{dln(1+T)} = \frac{dlny_H}{dln(1+T)}+\left(1-Y_e\frac{dlnY_x}{dln(1+T)}\right),\ \text{and} \tag{A.12}$$

$$\frac{dlny_{Hd}}{dln(1+T)} = \frac{dlny_H}{dln(1+T)}-Y_e\frac{dlnY_x}{dln(1+T)} = \frac{1-\beta}{\gamma-\beta}\frac{dlnQ}{dln(1+T)}-\frac{\gamma-1}{\gamma-\beta}Y_e\frac{dlnY_x}{dln(1+T)}. \tag{A.13}$$

From Eqs. (A.9), (A.10), and (A.11), we can easily see that $dlny_N/dln(1+T)$, $dlnr_N/dln(1+T)>0$ at $T=T^*=0$ when the home and foreign countries are symmetrical. From (35) and (A.9), we obtain the following equation:

$$\left(\frac{dlnQ}{dln(1+T)}+Y_e\frac{dlnY_x}{dln(1+T)}\right)\Bigg|_{T=T^*=0}=-\frac{\beta^2}{\gamma-\beta}\frac{1}{4\Delta^{TR}}\left(\begin{array}{c}\frac{(\gamma-1)^2}{\gamma-\beta}\left(2(1-R_H)+\beta\left(\frac{1}{1-\beta}+\frac{1}{\gamma-\beta}\right)\right)\\+\frac{\gamma-1}{\gamma-\beta}\left(\frac{(\gamma+\beta)(1-\beta)}{\gamma-\beta}+\gamma\right)R_H+\frac{\gamma(1-\beta)^2}{(\gamma-\beta)^2}{R_H}^2\end{array}\right)<0. \quad \text{(A.14)}$$

From Eqs. (A.10) and (A.14), $dlny_H/dln(1+T)$ and $dlnr_H/dln(1+T)<0$ at $T=T^*=0$. Finally, from Eqs. (35), (A.9), and (A.11)–(A.13), we know that $dlny_{He}/dln(1+T)<0$ and $dlny_{Hd}/dln(1+T)>0$ at $T=T^*=0$.

Q.E.D.

Proof of Proposition 7. From Eqs. (36) and (A.11), we know that $du_N/dln(1+T)>0$ at $T=T^*=0$. From Eqs. (37), (A.11), and (A.14), we obtain $du_H/dln(1+T)$ at $T=T^*=0$ when the home and foreign countries are symmetrical, as follows:

$$\frac{du_H}{dln(1+T)}\bigg|_{T=T^*=0}=-\frac{\beta^2(1-\beta)}{\gamma-\beta}\frac{r_H}{4\Delta^{TR}}\left(\begin{array}{c}(\gamma-1)\left(\frac{\gamma-1}{\gamma-\beta}\left(1-R_H+\frac{1}{1-\beta}+\frac{\beta}{\gamma-\beta}\right)+\frac{1}{\gamma-\beta}+\frac{(1-\beta)(\gamma+\beta)}{(\gamma-\beta)^2}R_H\right)\\+\frac{\gamma(1-\beta)^2}{(\gamma-\beta)^2}{R_H}^2-2\chi(\gamma-1)\left(\frac{\gamma-1}{\gamma-\beta}\left(\frac{1}{1-\beta}+\frac{\beta}{\gamma-\beta}\right)+\frac{\gamma(1-\beta)}{(\gamma-\beta)^2}R_H\right)\end{array}\right). \quad \text{(A.15)}$$

From Eq. (A.15), we obtain $du_H/dln(1+T)|_{T=T^*=0}<0$ when $\chi=1/2$. Since $R_H=1$ when $\chi=1$, we obtain $du_H/dln(1+T)$ at $T=T^*=0$ and $\chi=1$ as follows:

$$\frac{du_H}{dln(1+T)}\bigg|_{T=T^*=0}=\frac{R}{L}\frac{\beta^2(1-\beta)}{\gamma-\beta}\frac{\gamma-2+\beta}{1-\beta}\frac{1}{4\Delta^{TR}}>0. \quad \text{(A.16)}$$

From Eqs. (A.15) and (A.16), we obtain $du_H/dln(1+T)|_{T=T^*=0}<0$ when $\chi<\chi^+$, where $1/2<\chi^+<1$.

From Eqs. (35), (38), and (A.9), when the home and foreign countries are symmetrical, we obtain

$$\frac{dU}{dln(1+T)}\bigg|_{T=T^*=0}=\frac{\beta^2}{\gamma-\beta}\frac{\chi Lr_H}{4\Delta^{TR}}\left(\begin{array}{c}\frac{\beta(\gamma-1)}{\gamma-\beta}\left((\gamma-1)\left(1+\frac{1-\beta}{\gamma-\beta}\right)+\frac{(1-\beta)^2}{\gamma-\beta}R_H\right)\\+\frac{\gamma(1-\beta)^3}{(\gamma-\beta)^2}\left(\frac{(\gamma-1)\left((\gamma-\beta)^2-(\gamma-\beta^2)\right)}{\gamma(1-\beta)^2}-R_H\right)\end{array}\right)>0. \quad \text{(A.17)}$$

Q.E.D.

Proof of Proposition 8. Proposition 2 implies that $(1\text{-}\chi)L(-du_N/dln(1\text{-}t)|_{t=t^*=0})+\chi L(-du_H/dln(1\text{-}t)|_{t=t^*=0})=-dU/dln(1\text{-}t)|_{t=t^*=0}=-W$, where $W>0$. Proposition 7 implies that $(1\text{-}\chi)L(du_N/dln(1+T)|_{T=T^*=0})+\chi L(du_H/\ dln(1+T)|_{T=T^*=0})=dU/dln(1+T)|_{T=T^*=0}=Z$, where $Z>0$. Therefore, we obtain

$$-\frac{du_H}{du_N}\bigg|_{t=t^*=0}-\left(-\frac{du_H}{du_N}\bigg|_{T=T^*=0}\right)=\frac{\frac{du_N}{dln(1+T)}\Big|_{T=T^*=0}\left(\frac{1-\chi}{\chi}\left(-\frac{du_N}{dln(1-t)}\Big|_{t=t^*=0}\right)+\frac{W}{\chi L}\right)-\left(-\frac{du_N}{dln(1-t)}\Big|_{t=t^*=0}\right)\left(\frac{1-\chi}{\chi}\frac{du_N}{dln(1+T)}\Big|_{T=T^*=0}-\frac{Z}{\chi L}\right)}{(-du_N/dln(1-t)|_{t=t^*=0})(du_N/dln(1+T)|_{T=T^*=0})}$$

$$=\frac{Z\left(-du_N/dln(1-t)|_{t=t^*=0}\right)+W\left(du_N/dln(1+T)|_{T=T^*=0}\right)}{\chi L(-du_N/dln(1-t)|_{t=t^*=0})(du_N/dln(1+T)|_{T=T^*=0})}>0. \qquad \text{(A.18)}$$

Q.E.D.

Proof of Proposition 10. From Eq. (33), we obtain $dlnQ/dln(1+T)$ and $dlnY_x/dln(1+T)$ at $T=T^*=0$ when $dln(1+T)=dln(1+T^*)$ as follows:

$$\frac{dlnQ}{dln(1+T)}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=-\frac{R_H}{2(\gamma-1)}<0 \text{ and } \frac{dlnY_x}{dln(1+T)}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=-\frac{1}{1-\beta}<0\ . \qquad \text{(A.19)}$$

From Eqs. (A.11) and (A.19), we know that $dlnr_N/dln(1+T)$, $dlnr_H/dln(1+T)<0$ at $T=T^*=0$ and $dln(1+T)=dln(1+T^*)$.

As for the effects on the utilities of N-agents and H-agents, from Eqs. (36), (37), (A.11), and (A.19), we obtain

$$\frac{du_N}{dln(1+T)}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=\frac{\chi r_H}{2(\gamma-1)}\left((\gamma-1)-\frac{1-R_H}{1-\chi}(1-\beta)\right), \qquad \text{(A.20)}$$

$$\frac{du_H}{dln(1+T)}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=\frac{\chi r_H}{2(\gamma-1)}\left(\left(1-\frac{1}{\chi}\right)(\gamma-1)-\frac{R_H}{\chi}(1-\beta)\right)<0. \qquad \text{(A.21)}$$

Since $(1\text{-}R_H)/(1\text{-}\chi)=r_N/(L/R)<1$, we know that $du_N/dln(1+T)>0$ at $T=T^*=0$ and $dln(1+T)=dln(1+T^*)$. From Eqs. (A.20), (A.21), and $U=(1\text{-}\chi)Lu_N+\chi Lu_H$, we obtain

$$\frac{dU}{dln(1+T)}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=-\frac{(1-\beta)\chi r_H}{2(\gamma-1)}<0. \qquad \text{(A.22)}$$

Q.E.D.

Proof of Proposition 11. From Eqs. (24), (A.6), (A.7), (A.20), and (A.21), we obtain

$$-\frac{du_H}{du_N}\bigg|_{\substack{t=t^*=0\\dln(1-t)=dln(1-t^*)}}=-\frac{(\gamma-1)-\frac{R_H}{\chi}(\gamma-\beta)}{(\gamma-1)-\frac{1-R_H}{1-\chi}(\gamma-\beta)} \text{ and } -\frac{du_H}{du_N}\bigg|_{\substack{T=T^*=0\\dln(1+T)=dln(1+T^*)}}=-\frac{-\frac{1-\chi}{\chi}(\gamma-1)-\frac{R_H}{\chi}(1-\beta)}{(\gamma-1)-\frac{1-R_H}{1-\chi}(1-\beta)}. \qquad \text{(A.23)}$$

From Eq. (A.23), we obtain

$$-\frac{du_H}{du_N}\bigg|_{\substack{t=t^*=0\\ dln(1-t)=dln(1-t^*)}}-\left(-\frac{du_H}{du_N}\bigg|_{\substack{T=T^*=0\\ dln(1+T)=dln(1+T^*)}}\right)=\frac{(\gamma-1)(1-\beta)\frac{1-R_H}{1-\chi}}{\chi\left((\gamma-1)-\frac{1-R_H}{1-\chi}(\gamma-\beta)\right)\left((\gamma-1)-\frac{1-R_H}{1-\chi}(1-\beta)\right)}>0. \quad \text{(A.24)}$$

Q.E.D.

Proof of Proposition 12. From comparative statics matrix (43), we obtain

$$-\frac{dlnQ}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}=-\frac{\beta^2}{(\gamma-\beta)^2}\frac{R_H}{2\Delta^{TX}}\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(1+\beta+\frac{\beta(1-\beta)}{\gamma-\beta}\right)\right)<0, \quad \text{(A.25)}$$

$$-\frac{dlnY_x}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}=\frac{\beta^3(\gamma-1)}{(\gamma-\beta)^3}\frac{1}{\Delta^{TX}}\left((1-\beta)R_H+(\gamma-1)\right)>0. \quad \text{(A.26)}$$

From Eqs. (41), (42), (A.25), and (A.26), we obtain the effects of increasing t_H on u_N, u_H, and U as follows:

$$-\frac{du_N}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}=\frac{\beta^2(\gamma-1)}{(\gamma-\beta)^2}\frac{\chi r_H}{2\Delta^{TX}}\begin{pmatrix}\left(1-\frac{1-R_H}{1-\chi}\frac{1-\beta}{\gamma-1}\right)\left(2(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H\right)\\ +\left(R_H+\frac{1-R_H}{1-\chi}\right)\frac{\gamma(1-\beta)^2}{\gamma-\beta}\end{pmatrix}>0, \quad \text{(A.27)}$$

$$-\frac{du_H}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}=-\frac{\beta^2(1-\beta)}{(\gamma-\beta)^2}\frac{r_H}{2\Delta^{TX}}\begin{pmatrix}\frac{(\gamma-1)(1-\chi)}{1-\beta}\left(1-\frac{1-R_H}{1-\chi}\frac{1-\beta}{\gamma-1}\right)\left(2(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H\right)\\ +(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}(2-R_H)+\frac{\gamma(1-\beta)}{\gamma-\beta}(1-\chi R_H)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H\right)\end{pmatrix}<0, \quad \text{(A.28)}$$

$$-\frac{dU}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}=-\frac{\beta^2(1-\beta)}{(\gamma-\beta)^2}\frac{\chi r_H}{2\Delta^{TX}}\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}(2-R_H)\right)\right)=-V<0. \quad \text{(A.29)}$$

From Eqs. (A.3) and (A.27), we obtain

$$-\frac{du_N}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}-\left(\frac{du_N}{dln(1-t)}\bigg|_{t=t^*=0}\right)$$

$$=\frac{\beta^2}{(\gamma-\beta)^2}\frac{r_N}{2\Delta^{TX}}\begin{pmatrix}2\chi(\gamma-1)\left((\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H\right)\\ +(1-\beta)(1-R_H)\left(2(\gamma-1)\left(1+\frac{\beta(1-\beta)}{\gamma-\beta}\right)+\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H\right)\end{pmatrix}>0. \quad \text{(A.30)}$$

Moreover, from Eqs. (19), (20), (25), and (A.29), we obtain

$$W-V=\frac{dU}{dln(1-t)}\bigg|_{t=t^*=0}-\frac{dU}{dln(1-t_H)}\bigg|_{t_H=t_H{}^*=0}$$

$$=\frac{\beta^2(1-\beta)}{(\gamma-\beta)^2}\frac{(1-\chi)Lr_N}{2\Delta^{TX}}\left(\frac{\gamma(1-\beta)^2}{\gamma-\beta}R_H+(\gamma-1)\left(2+\frac{\beta(1-\beta)}{\gamma-\beta}(2-R_H)\right)\right)>0. \quad \text{(A.31)}$$

Therefore, we obtain

$$-\frac{du_H}{du_N}\bigg|_{t=t^*=0}-\left(-\frac{du_H}{du_N}\bigg|_{t_H=t_H{}^*=0}\right)$$

$$=\frac{-\frac{du_N}{dln(1-t)}\Big|_{t=t^*=0}\left(-\frac{1-\chi}{\chi}\left(-\frac{du_N}{dln(1-t_H)}\Big|_{t_H=t_H{}^*=0}\right)-\frac{V}{\chi L}\right)-\left(-\frac{du_N}{dln(1-t_H)}\Big|_{t_H=t_H{}^*=0}\right)\left(-\frac{1-\chi}{\chi}\frac{du_N}{dln(1-t)}\Big|_{t=t^*=0}-\frac{W}{\chi L}\right)}{\left(-du_N/dln(1-t)|_{t=t^*=0}\right)\left(-du_N/dln(1-t_H)|_{t_H=t_H{}^*=0}\right)}$$

$$= \frac{W\left(-du_N/dln(1-t_H)|_{t_H=t_H{}^*=0}\right)-V\left(-du_N/dln(1-t)|_{t=t^*=0}\right)}{\chi L\left(-du_N/dln(1-t)|_{t=t^*=0}\right)\left(-du_N/dln(1-t_H)|_{t_H=t_H{}^*=0}\right)} > 0. \tag{A.32}$$

To compare the equality improvement costs of income tax levies on the exporter and the import tariff, we obtain the following equation:

$$-\frac{du_H}{du_N}\bigg|_{t_H=t_H{}^*=0} - \left(-\frac{du_H}{du_N}\bigg|_{T=T^*=0}\right) = \frac{Z\left(-du_N/dln(1-t_H)|_{t_H=t_H{}^*=0}\right)+V\left(du_N/dln(1+T)|_{T=T^*=0}\right)}{\chi L\left(-du_N/dln(1-t_H)|_{t_H=t_H{}^*=0}\right)\left(du_N/dln(1+T)|_{T=T^*=0}\right)} > 0\ . \tag{A.33}$$

Q.E.D.

Proof of Proposition 13. From the comparative statics matrix (43), we obtain

$$-\frac{dlnQ}{dln(1-t_H)}\bigg|_{\substack{t_H=t_H{}^*=0 \\ dln(1-t_H)=dln(1-t_H)}} = -\frac{R_H}{\gamma-1} < 0 \text{ and } -\frac{dlnY_x}{dln(1-t_H)}\bigg|_{\substack{t_H=t_H{}^*=0 \\ dln(1-t_H)=dln(1-t_H)}} = 0. \tag{A.34}$$

From Eqs. (41), (42), (A.23), (A.24), and (A.34), we obtain

$$-\frac{du_H}{du_N}\bigg|_{\substack{t_H=t_H{}^*=0 \\ dln(1-t_H)=dln(1-t_H)}} = -\frac{-\frac{1-\chi}{\chi}(\gamma-1)-\frac{R_H}{\chi}(1-\beta)}{(\gamma-1)-\frac{1-R_H}{1-\chi}(1-\beta)} = -\frac{du_H}{du_N}\bigg|_{\substack{T=T^*=0 \\ dln(1+T)=dln(1+T^*)}}. \tag{A.35}$$

Q.E.D.

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