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Efficiency in a federation with public input provision

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Abstract

We examine efficiency in a federal system in which different tiers of government provide public inputs and their tax bases overlap in two taxation models: unit labour tax financing and an ad-valorem labour income tax financing. In contrast to the findings of the previous literature, we show that the federal government can internalize vertical fiscal externalities and satisfy the efficiency rules for public inputs even if it chooses a positive tax rate in the case of a unit tax. On the other hand, the federal government uses a labour income subsidy to achieve the second-best allocation, however, it can replicate the government in a unitary jurisdiction without a negative inter-governmental transfer also in the case of an ad-valorem tax.

JEL classification: H71; H72; H77.

Keywords: vertical fiscal externalities; public input provision; unit labour tax; ad-valorem labour income tax

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1 Introduction

Following the seminal remarks by Boadway and Keen (1996), who considered the model of public goods provision by different tiers of government and pointed out that the federal government should choose a negative tax rate as the subsidy for the common tax base in order to internalize a vertical tax externality (thus requiring negative inter-governmental transfers from the states to finance the federal public good), one of the issues in the literature on vertical fiscal externalities has been the sign of the federal tax rate and the direction of transfers in various theoretical models. One stream considers asymmetry or migration between states.[1] The other stream applies different instruments of taxation and the type of public expenditure.[2]

Kotsogiannis and Martínez (2008) assume that both tiers of government use ad-valorem labour income taxes instead of the unit tax in the Boadway–Keen (BK) model. According to their main results, since the federal tax revenue by ad-valorem taxation includes not only a shrinkage of employment but also a rise in the wage rate as a state tax increases, a positive vertical externality can be seen if the demand for labour is inelastic. Thus, the federal government can choose a positive tax rate and positive transfers to the states in that case. Dahlby and Wilson (2003) and Martínez (2008) assume that states provide public inputs and the federal government provides the public good in an ad-valorem tax model. The main remarks from their analyses are that vertical fiscal externalities by tax-setting and public input provision are independent, and thus, the matching grant is needed to internalize the

[1] See, for example, Boadway et al. (1998) and Sato (2000).

[2] The arguments about applying different types of taxation or public expenditure have also been made in analyses of horizontal externalities in terms of whether public input is over- or under-provided in capital tax competition (e.g., Zodrow and Mieszkowski [1986], Noiset [1995] and Matsumoto [1998]) and whether unit tax or ad-valorem tax causes more miserable tax competition that brings about more inefficiently lower level of public good (e.g., Lockwood [2004] and Akai et al. [2011]).

vertical externality by public input.[3]

This paper aims to investigate public input provision by unit labour tax financing and ad-valorem labour income tax financing in a model of vertical fiscal externalities. We firstly consider the model of joint provision of public inputs financed by unit labour taxes, which has a characteristic of counter-setting to the BK model with respect to the type of public expenditure. Then, we extended the model to the ad-valorem labour income taxation, which is the joint provision of public inputs version of the Dahlby–Wilson–Martínez (DWM) model.

One of the purposes of this paper is to consider the dominance issue of the vertical externalities by tax and public input by extending the BK model to the public input version, as state public input is considered to cause a positive externality on the federal budget while state tax brings about a negative externality. Productivity enhancing public inputs can be considered to increase employment as a result of enhancing the demand for labour in contrast to consumable public goods in the BK model, which affect only the household utility. Hence, it is supposed that the conditions for, and thus the results of, optimization behaviour of the federal and state governments are different from those in the BK model.

Another purpose of this paper is to re-consider the mechanism of vertical fiscal externalities in the model of ad-valorem tax-financed public input. Although Dahlby and Wilson (2003) and Martínez (2008) have already studied the model of public input, the vertical externalities by the state tax and public input are impossible to compare because of unobvious signs that highly depend on the properties of the production function in their model. As

[3]Although Wrede (2000) and Madies (2008) are also cited in the literature on vertical fiscal externalities with public input, they are different from ours with respect to a framework because they assume a Nash game between the federal government and states.

mentioned above, the federal tax revenue by ad-valorem taxation includes changes in both employment and the wage rate. The wage rate always rises irrespective of whether the tax rate or public input increases, while employment increases with public input and inversely moves as the state tax rate rises. Consequently, the relation between the vertical externalities of the state tax and public input has been considered to depend on the degree of the rise of the wage rate, which is influenced by the elasticities of labour demand and supply in the literature.

To avoid such complications, Dahlby and Wilson (2003) have discussed only on the optimal condition for the state government's policy without solving the maximization problem of the federal government and suggested a necessity of a matching grant as an additional federal policy instrument to internalize the vertical externality, which means that the federal government cannot achieve an efficient result by existing policy instrument in the model of ad-valorem tax. Martínez (2008) has imposed a specific condition that requires each of vertical externality of state tax and state public input to be respectively internalized on the solution of the maximization problem of the federal government. However, the implication is inconsistent with the assumption as we will explain later in the paper.

In contrast to these previous literature, we comprehensively characterize the equilibrium policy variables in the Stackelberg game which is generally employed in the literature on vertical fiscal externalities, by solving the maximization problems of the federal and state governments without any specific assumptions. The neutrality theorem of state public input which is derived from the investigation on the model of unit labour tax financing is significantly helpful to understand the mechanism of the vertical fiscal externalities in the model of ad-valorem tax financing; that is, it shows that the state public input is neutral to employment irrespective of whether it is financed by a unit labour tax or an ad-valorem labour income tax.

We show that the federal government can internalize vertical fiscal externalities and satisfy the efficiency rules for public inputs even if it chooses a positive tax rate in the model of unit labour tax financing, which contrasts with the remarks concerning the BK model. Furthermore, the second-best rules for public expenditure by the tiers of government with a positive federal tax can be achieved irrespective of the type of federal expenditure if only the states provide public inputs by unit labour tax financing.

On the other hand, in the model of ad-valorem tax financing, we present that the federal government internalizes vertical fiscal externalities by using a labour income subsidy which is comparable to the matching grant mentioned by Dahlby and Wilson (2003), and, furthermore, that it does not need negative inter-governmental transfers from the states. In contrast to the DWM model, our results do not require an additional policy instrument to internalize the vertical externalities and a specific condition for solving the optimization problem.

The remainder of this paper is organized as follows. We outline the model of vertical fiscal externalities with public input provision by unit labour tax financing in section 2 and characterize the equilibrium in the situation of separated government structure in section 3. Then, we consider the model of ad-valorem labour income tax financing in section 4. Finally, we provide concluding remarks in section 5.

2 Basic Model

In line with a common method of study of vertical fiscal externalities, we construct a model in which two tiers of government use labour taxation in order to provide public inputs. The model here is basically same as the Boadway-Keen (BK) model except utilizing public inputs instead of public goods. There are symmetric and small $k(> 1)$ states in the federation. A representative household in each state has utility of the form $u(x, l)$, which has the usual properties; that is, it is quasi-concave, increasing in x, and decreasing in l, where x is a private good (and numeraire) and l is labour. Since interstate migration is not assumed in this paper, the number of household is normalized to one.

The public input by a state government, denoted by e, is financed by taxation on labour l at rate t. The federal public input E is also financed by labour taxation at rate T. The consolidated tax rate is denoted by $\tau \equiv t+T$.

The representative household maximizes $u(x, l)$ subject to the budget constraint $x = (w-\tau)l$, where w is the gross wage rate and the net wage rate $(w-\tau)$ is denoted by $\overline{w}$. Labour supply, denoted by $l(\overline{w})$, is implicitly defined by the first-order condition of household maximization. It is assumed that $l'(\overline{w}) > 0$.[4] Indirect utility, given by $v(\overline{w}) = u(\overline{w}l, l)$, derives an envelope property, $v' = u_x l$.

Output is produced in each state with production technology $f(h, l, p)$, which has properties $f(h, 0, p) = 0$, $f_i > 0$, $f_{ii} < 0$, and $f_{ij} = f_{ji} > 0$, $\forall i, j = h, l, p$. Output can be transformed into x, e and E without additional cost. Since h is assumed as a fixed factor (e.g. land) and to be owned by private firms, the profits appear in the private sector. The public input in each state is denoted as p and is jointly provided by the state and the federal government with quasi-concave and homothetic properties: $p = p(E, e)$. For example, we imagine a road system (p) in a jurisdiction organized by interstate highways (E) and state roads (e). E is assumed to be the pure

[4]For this inequality, two assumptions are needed: an additive-separability of utility function and a linearity of the partial utility concerning x.

public input between states; that is, its increase everywhere contributes to production in every state by improving the convenience of road transport. We assume that p is a factor-augmenting public input, and thus define production technology as constant return to scale in private factors only;[5] that is:

$$f = f_h h + f_l l,\ f_p = f_{hp} h + f_{lp} l. \tag{1}$$

The private sector competitively maximizes profits, given by $\pi = f(l, p) - wl$, and chooses a demand for labour that satisfies $f_l(l, p) = w$. Since a labour market is assumed to satisfy the condition of a perfectly competitive equilibrium, w is given by $w = w(\tau, p)$. Let $z \equiv \frac{wl'}{l}(> 0)$ denote the elasticity of the labour supply with respect to the gross wage rate and let $\epsilon \equiv \frac{f_l}{lf_{ll}}(< 0)$ denote the elasticity of the demand for labour; then, we obtain $w_\tau = \frac{z}{z-\varepsilon} > 0$, $\overline{w}_\tau = w_\tau - 1 = \frac{\epsilon}{z-\epsilon} < 0$, and $w_p = -\frac{\epsilon}{z-\epsilon} f_{lp} > 0$, where the inequalities follow from $w > 0$ and $f_{lp} > 0$. From these marginal impacts of policy variables on the wage rate, we obtain the following relation for later use:

$$-\frac{w_p}{\overline{w}_\tau} = f_{lp}. \tag{2}$$

From the total differential of the indirect utility function with respect to τ and p, we recognize that the left hand side of (2) represents a marginal rate of substitution (MRS) between the burden of consolidated tax and the benefit from jointly provided public input regarding indirect utility. Eq. (2) shows the MRS is equalized to the marginal contribution of public input on the

[5] See Feehan (1989) for detailed consideration about the type of public input. Although we examined the model with unpaid-factor type public inputs, following Feehan and Batina (2007), the main result was same as that in the model with factor-augmenting type. Hence, we do not mention the type of public input in the rest of this paper.

marginal productivity of labour by optimization behaviours of households and private firms.

In the same manner as the BK and the Dahlby–Wilson–Martínez (DWM) models, the profits π are assumed to be taxed by the federal government at rate θ and by the state at rate $(1-\theta)$, where $0 \leq \theta \leq 1$.[6] The marginal impacts of τ and p on π can be derived as $\pi_\tau = -w_\tau l < 0$ and $\pi_p = f_p - w_p l > 0$, respectively.[7]

Before the analyses, we consider the benchmark obtained in a unitary jurisdiction where the government maximizes $kv(\overline{w})$ by choosing τ and two public inputs subject to the consolidated budget constraint $E + ke = k(\tau l + \pi)$. From the first-order conditions, we obtain

$$f_p p_E = \frac{1}{k}, \tag{3}$$

$$f_p p_e = 1. \tag{4}$$

Eqs. (3) and (4) show the second-best rules for federal public input E and state public input e, respectively. These equations seem to follow Kaizuka's (1965) condition in a federal system; however, the quantities of E and e are not same as those in the case of a lump-sum tax.[8]

[6] The reasons why we still assume that 100% of the profits are acquired by public sector even though this assumption is strong and unreal are the follows. First, a main objective of this paper is to find whether our results will be different from those particularly in the BK model, by utilizing the assumption on the provision of public inputs. Second, under the assumption of a controllable profit tax by both tiers of government, it is easily supposed that they choose only the profit tax whose incidence falls on a fixed production factor h in order to avoid the shrinkage of a labour tax base.

[7] Using (1) and $w_p = -\frac{\epsilon}{z-\epsilon} f_{lp}$, and noticing that $0 < \left|\frac{\epsilon}{z-\epsilon}\right| < 1$, one can confirm that $\pi_p = f_p - w_p l > 0$.

[8] Feehan and Matsumoto (2002) discuss the production efficiency of public inputs financed by a distortionary tax in a general equilibrium model. Similar to their indication, the first-order condition for τ is derived as $\frac{k\eta}{u_x} = \frac{1}{\left(1-\frac{\tau l'}{l}\right)}$ in our model, where η denotes the Lagrangian multiplier. The right hand of this condition includes the marginal cost of

Then, we summarize the budget constraints of the state and the federal government. Denoting by S an inter-governmental transfer, each of them is given as follows:

$$
\begin{aligned}
tl + (1-\theta)\,\pi + S - e &= 0 \equiv \Psi, &(5)\\
Tl + \theta\pi - S - \frac{E}{k} &= 0. &(6)
\end{aligned}
$$

Note that the state budget constraint is denoted by Ψ as an implicit function for later use.

Denoting by R the federal tax revenue, the marginal impacts of the state policy variables on the federal tax revenue are summarized as follows:

$$
\begin{aligned}
R_t &= Tl'\overline{w}_\tau + \theta\pi_\tau, &(7)\\
R_e &= \left[Tl'w_p + \theta\pi_p\right] p_e. &(8)
\end{aligned}
$$

Similar to the remark by Boadway and Keen (1996) on a unit labour tax, we assume that a negative vertical tax externality; that is, the sign of (7) is also assumed to be negative in our model. On the other hand, according to (8), the budgetary impact of state public input is positive.

The main contrast between our paper and the previous literature is the aim of the federal government's policy toward these vertical externalities. In other words, while the federal government is assumed to eliminate vertical externalities in the literature; that is, $R_t = 0$ in Boadway and Keen (1996) and $R_t = R_e = 0$ in Martínez (2008), our model does not need such a

public funds (MCPF) with respect to a shrinkage of employment. Therefore, we call (3) and (4) the second-best rules, not the first-best rules mentioed by Kaizuka (1965).

condition for solving the optimization problem of the federal government.

3 Optimal policies in a separated government

Next, we consider the optimal policies in the case of a separated government in a basic model. In line with a basic scenario of vertical fiscal externalities, we assume the federal government as a Stackelberg leader to choose its policy variables at the first stage and the state governments as followers to choose t and e at the second stage. Since we solve the game by backward induction, we begin to characterize the optimization problem of the follower.

3.1 The state's problem

A representative state as a follower ignores the impacts of its decision on the federal budget described by (7) and (8), and thus chooses t and e to maximize $v(\overline{w})$ subject to (5), taking T, E, S, and θ as given. Denoting by μ the Lagrangian multiplier of the state's optimization, the first-order conditions for t and e are derived respectively as follows:

$$v'\overline{w}_\tau + \mu\Psi_t = 0, \tag{9}$$

$$v' w_p p_e + \mu\Psi_e = 0. \tag{10}$$

Combining them derives

$$\frac{\Psi_e}{\Psi_t} = \frac{w_p}{\overline{w}_\tau} p_e. \tag{11}$$

Using (1), (2) and (11), we obtain

$$
\begin{aligned}
f_p p_e &= 1 + \theta \left(f_p - f_{lp} l \right) p_e \\
&= 1 + \theta f_{hp} h p_e. \qquad (12)
\end{aligned}
$$

Comparing (12) to (4), we recognize that state public input is underprovided, because the sign of the second term on the right hand side of (12) is positive. This term directly represents the federal government's share of the profit gain for a fixed factor yielded by the state public input, and the marginal cost of public funds (MCPF) for the state.

To consider why the term appears in the optimal condition for the state public input, we use the following relation which is derived from a total differential of (5) with respect to t and e subject to a balanced budget rule.

$$
dt = -\frac{\Psi_e}{\Psi_t} de \qquad (13)
$$

Denoting by $l' \overline{w}_\tau dt + l' w_p p_e de$ a total impact of state tax and state public input on employment and inserting (11) and (13) into the foregoing impact, we obtain

$$
l' \overline{w}_\tau dt + l' w_p p_e de = l' \overline{w}_\tau \left(-\frac{w_p}{\overline{w}_\tau} p_e de \right) + l' w_p p_e de = 0. \qquad (14)
$$

Eq. (14) gives the following neutrality theorem.

Theorem *A unit labour tax-financed state public input is neutral to employment.*

In a diagrammatic image of a labour market, the first term on the left hand side of (14) represents that the labour supply curve shifts to left in response to a rise in state tax, while the second term on the LHS of (14) means the shift of the labour demand curve to right in response to an increase in state public input. Consequently, the impacts of state tax and state public input on employment can completely offset each other by the optimization behaviour of the state government, and thus the only rise in the gross wage rate is remained a labour market.

This neutrality theorem, which is a distinctive feature of the model, brings about an interesting result as the following. Using (2), (11), (13), and (14), the total differential of the federal tax revenue R with respect to t and e gives

$$\begin{aligned} R_e de + R_t dt &= \theta \left(\pi_p + f_{lp}\pi_\tau\right) p_e de \\ &= \theta \left(f_p - f_{lp} l\right) p_e de \\ &= \theta f_{hp} h p_e de. \end{aligned} \tag{15}$$

From (15), we understand the meaning of the second term on the RHS of (12) as the following proposition.

Proposition 1 *The vertical fiscal externalities by state tax and public input are reduced to the increase in the federal profit tax revenue in the model where the state government provides public input by unit labour tax financing.*

While, in the previous literature, a vertical externality is mainly imagined as the shrinkage of employment caused by state government taxation, which is ignored by the state, the impact of taxation on employment is offset by that of public input in the model of state public input provision as discussed above. This means that the federal government does not have to consider an additional distortion caused by the reaction of the state in a labour market in the maximization problem of the federal government.

On the other hand, we confirm from (15) that an increase in the profit, and thus the federal profit tax revenue if $\theta > 0$, is happened as a result of the optimization behaviour of the state. Eq. (12) shows that the state treats such a partially shifting of the profit gain to the federal government revenue as the MCPF because the state cannot be aware that it brings about an increase in the federal public input which contributes production of the private sector in the state.

This evidence can be seen only in the model of state public input provision, not in the BK model in which the state public good still has a distortional effect on employment even though it is financed by unit labour tax.[9]

3.2 The strategy of the federal government

We next consider the maximization problem of the federal government, returning to the first stage. While it is obvious from (12) that the second-best rule for e can be achieved as $\theta = 0$ is exogenously assumed, we have an interest in whether the federal government can endogenously choose $\theta = 0$ as the optimal profit tax rate to internalize vertical fiscal externalities and what the sign of the federal labour tax is.[10]

Transforming (12) into $\theta f_{lp} l p_e + (1-\theta) f_p p_e - 1 = 0$ and denoting it by Ω as an implicit function, we obtain following two-equation system by the total differentials of Ω and Ψ with respect to the federal and the state variables.

[9] This can be confirmed by (23) in Boadway and Keen (1996).

[10] Although the federal government in the BK model might be able to choose θ endogenously, the equilibrium federal tax rate must be zero in the case because a vertical tax externality cannot be eliminated only by setting θ to be zero. Therefore, negative intergovernmental transfers are still required to finance the federal public good in the case of the BK model. See p. 147 of Boadway and Keen (1996).

$$\begin{pmatrix} \Omega_t & \Omega_e \\ \Psi_t & \Psi_e \end{pmatrix} \begin{pmatrix} dt \\ de \end{pmatrix} = - \begin{pmatrix} \Omega_T & \Omega_E & \Omega_S & \Omega_\theta \\ \Psi_T & \Psi_E & \Psi_S & \Psi_\theta \end{pmatrix} \begin{pmatrix} dT \\ dE \\ dS \\ d\theta \end{pmatrix}$$

From standard procedures for comparative statics, the reactions of the state to the federal policy variables are derived as follows.

$$\begin{aligned} t_T &= -1 + \Lambda \Omega_e l,\ t_E = \Lambda \left(\Omega_E \Psi_e - \Omega_e \Psi_E \right), \\ t_S &= -\Lambda \Omega_e,\ t_\theta = \Lambda \left(\Omega_\theta \Psi_e - \Omega_e \Psi_\theta \right), \\ e_T &= -\Lambda \Omega_t l,\ e_E = \Lambda \left(\Omega_t \Psi_E - \Omega_E \Psi_t \right), \\ e_S &= \Lambda \Omega_t, \text{ and } e_\theta = \Lambda \left(\Omega_t \Psi_\theta - \Omega_\theta \Psi_t \right), \end{aligned} \tag{16}$$

where $\Lambda = \frac{-1}{\Omega_t \Psi_e - \Omega_e \Psi_t}$, which has to be negative for the stability of the system. In the manipulation, we use $\Omega_t = \Omega_T$, $\Psi_t = \Psi_T + l$, $\Omega_S = 0$, and $\Psi_S = 1$. While we cannot identify the sign of each reaction of the state in (16), we at least know that each sign does not become zero without very specific conditions.

The federal government chooses T, E, S, and θ to maximize $kv(\overline{w})$ subject to (6) and (16). Denoting by $\Delta \equiv v(\overline{w}) + \lambda \left[Tl + \theta\pi - S - \frac{E}{k} \right]$ the federal government's Lagrangian function, we obtain the following first-order conditions for policy variables, respectively:

$$\Delta_\tau (1+t_T) + \Delta_p p_e e_T + \lambda l = 0, \tag{17}$$

$$\Delta_\tau t_E + \Delta_p (p_e e_E + p_E) - \frac{\lambda}{k} = 0, \tag{18}$$

$$\Delta_\tau t_S + \Delta_p p_e e_S - \lambda = 0, \tag{19}$$

$$\Delta_\tau t_\theta + \Delta_p p_e e_\theta + \lambda \pi = 0, \tag{20}$$

where $\Delta_\tau \equiv v_w \overline{w}_\tau + \lambda [Tl' \overline{w}_\tau + \theta \pi_\tau]$ and $\Delta_p \equiv v_w w_p + \lambda [Tl' w_p + \theta \pi_p]$.

From the first (t_T), third (t_S), fifth (e_T), and seventh (e_S) equalities in (16), we derive $1 + t_T = -t_S l$ and $e_T = -e_S l$. Using them, we find that eqs. (17) and (19) are completely equivalent, that is, the following remark is obtained.

Remark 1 *S is not compatible policy variable with T in the current model.*

Remark 1 means that the first-order condition for the inter-governmental transfer is redundant in the optimization problem of the federal government in our model. This is an essential difference between our model and the BK model.

In the BK model, where both tiers of government provide public goods, the federal government has to consider two types of distortional effect of its labour tax on employment; the one caused by itself and the other via a reaction of the state. On the other hand, an inter-governmental lump-sum transfer brings only an income effect on the state's budget. Therefore, the first-order conditions for the federal labour tax and the transfer are not equivalent, and thus a negative inter-governmental transfer is needed to avoid the distortion of the federal labour tax, in particular, which is caused by the state's reaction, and to finance federal public good in the BK model.[11]

[11] Applying our manipulation to the solution in the BK model, one can confirm that

In contrast, the federal government does not have to consider such a distortion via the reaction of the state in a labour market as above-discussion on proposition 1.[12] Therefore, the negative inter-governmental transfer is not needed for financing the federal public input in our model.

For this reason, the optimization problem of the federal government can be reduced to (17), (18), and (20). After algebraic manipulation, these equations are transformed into the follows.

$$
\begin{aligned}
l\Phi + \lambda\Psi_t \left(f_p p_e - 1\right) e_T &= 0, &(21)\\
-\frac{p_E}{p_e}\Phi + \lambda\Psi_t \left[\left(f_p p_e - 1\right) e_E + f_p p_E - \frac{1}{k}\right] &= 0, &(22)\\
\pi\Phi + \lambda\Psi_t \left(f_p p_e - 1\right) e_\theta &= 0, &(23)
\end{aligned}
$$

where $\Phi \equiv v'\overline{w}_T + \lambda\left[l + (t+T)\, l'\overline{w}_T + \pi_T\right]$.

From (21), (22), and (23), we obtain the following proposition.

Proposition 2 *In the model where both tiers of government jointly provide public inputs by unit labour tax financing, the federal government can internalize vertical fiscal externalities and replicate the government in a unitary jurisdiction by devolving the profit tax to the state.*

Proof. Eqs. (21) and (23) derive $\Phi = 0$ and $f_p p_e = 1$, that is, $\theta = 0$. Using them, we obtain $f_p p_E = \frac{1}{k}$ from (22). ■

Direct effects of tax-setting and public input provision by the federal government on employment can offset each other in the same manner as the

$1 + t_T \neq -t_S l$ and $g_T \neq -g_S l$, where g denotes a state public good.

[12] Notice that the above-mentioned equalities $1+t_T = -t_S l$ and $e_T = -e_S l$ do not mean that the effects of the federal labour tax and the negative transfer on the state's behaviour are parallel but that they are indifferent for the federal government's Lagrangian function.

state's optimization shown by the neutrality theorem. In addition, $\theta = 0$ derives an efficient level of state public input. Therefore, the federal government can achieve the second-best allocation by relinquishing control of the profit tax.

Furthermore, we find that the pair of equilibrium conditions, $\theta = 0$ and $\Phi = 0$, leads the following.

Proposition 3 *The federal government sets its labour tax rate T to be positive as $\theta = 0$ and $\Phi = 0$, which are the equilibrium conditions of the Stackelberg game where the federal government behaves as the leader and state governments are followers.*

Proof. Noting that we assume the absence of the Laffer effect in the same manner as the previous literature; that is, it is assured for both tiers of government that a rise in its own tax rate can increase its own tax revenue despite of the shrinkage of employment. This means that $\Psi_t > 0$ and $\mu > 0$ from (9). Since $l + (t+T)\, l'\overline{w}_\tau + \pi_\tau$ in the square brackets on Φ is equivalent to the impact of tax rate change on the tax revenue of the government in a unitary jurisdiction,the sign of this term is positive. Thus, $\Phi = v'\overline{w}_\tau + \lambda \left[l + (t+T)\, l'\overline{w}_\tau + \pi_\tau\right] = 0$ derives $\lambda > 0$ because the sign of $v'\overline{w}_\tau$ is defined as negative. Then, inserting (9) into Φ, we rewrite $\Phi = 0$ as $(\lambda - \mu)\, \Psi_t + \lambda \left(Tl'\overline{w}_\tau + \theta\pi_\tau\right) = 0$. Since we consider the situation of a negative vertical tax externality in order to compare our result with that in the BK model, the sign of the second term of the foregoing equation is confirmed to be negative by referring to (7). Then, we obtain that $\lambda - \mu > 0$. When the federal government chooses $\theta = 0$, it is derived that $\Phi|_{\theta=0} = (\lambda - \mu)\, \Psi_t + \lambda T l'\overline{w}_\tau = 0$. Consequently, we find that the optimal federal tax rate in this case is described as $T = \frac{-1}{\lambda l'\overline{w}_\tau}(\lambda - \mu)\, \Psi_t > 0$.[13] ■

[13] It would be slightly unusual that the optimal federal tax rate includes the Lagrangian

We can confirm that $t > 0$ at the equilibrium as the follows. First, we obtain that $\Psi_t|_{\theta=0} = l + tl'\overline{w}_\tau + \pi_\tau > 0$ under the condition $\theta = 0$. It derives $t = -\frac{1}{l'\overline{w}_\tau}(\pi_\tau + l)$. Then, remembering that $\pi_\tau = -w_\tau l$ and $\overline{w}_\tau = w_\tau - 1$, we obtain that $t = \frac{l}{l'} > 0$. Therefore, we recognize that both of the labour tax rates of the federal and state governments are positive in the model.

Our proposition 3 does not require the assumption on the elasticities of labour demand and supply which is most important to clarify the sign of vertical externality in Kotsogiannis and Martínez (2008). Furthermore, as mentioned by remark 1, an inter-governmental transfer is redundant policy instrument in spite of a negative vertical tax externality in our model, in contrast to the BK model.

3.3 Discussion with extensions

We obtain contrast remarks to those in Boadway and Keen (1996) by the opposite model setting about government expenditure from theirs; that is, the both tiers of government provide public inputs while the case of public goods provision is considered in the BK model. However, it is somewhat unobvious which modelling is significant; state's public input or the federal public input. Therefore, we next discuss what results are obtained from some extensions of the basic model: the case of divided responsibility between the federal and state governments in which the federal government provides public good while the state government still provides public input, on the other hand, the case of dual provider in which the state government is assumed provide public good and public input whereas the federal government provides public input only.

multipliers. However, we do not expand it further, because it is sufficient for proposition 3 that the sign of the optimal tax rate is positive.

3.3.1 Division of responsibilities

The responsibility of each tier of government for public expenditure is divided into public good and public input. We briefly describe different aspects of the basic model in section 2. First, a household's utility function is formed by $u(x,l)+B(G)$, where $B(\cdot)$ is concave and increasing in G, which is the pure public good provided by the federal government. Then, although the properties of the production function are equivalent to those in the previous model, public input p consists only of e in this model. The budget constraint of the federal government slightly changes to $Tl+\theta\pi-S-\frac{G}{k}=0$ with G instead of E. In this model, the second-best rule for G is represented as $\frac{kB'}{u_x}=\frac{1}{1-\frac{\tau l'}{l}}$, while eq. (4) is again the rule for e.

We again solve the Stackelberg game where the federal government chooses T, G, S and θ at the first stage and the state government chooses t and e at the second stage. By backward induction, we start to solve the problem at the second stage. Since the optimization problem of the state is unchanged, the first-order conditions are again represented by (9) and (10), and thus eq. (12) is also derived. However, at the first stage, the following new condition for G is derived instead of (18) in addition to (17), (19), and (20) from the maximization of $k[v(\overline{w})+B(G)]$ by choosing T, G, S and θ subject to the foregoing new budget constraint and state's reaction.

$$B'-\frac{\lambda}{k}=0 \tag{24}$$

As a result, the system of the first-order conditions can be reduced as follows.

$$
\begin{aligned}
l\Phi + kB'\Psi_t\left(f_p p_e - 1\right) e_T &= 0, \\
\pi\Phi + kB'\Psi_t\left(f_p p_e - 1\right) e_\theta &= 0.
\end{aligned}
$$

where $\Phi \equiv v'\overline{w}_\tau + \lambda\left[l + (t+T)\, l'\overline{w}_\tau + \pi_\tau\right]$ again.

We recognize that $\theta = 0$ and $\Phi = 0$ are the equilibrium conditions again. Then, inserting (24) and $\theta = 0$ into Φ, we obtain the second-best rule mentioned above under $\Phi = 0$. In addition, inserting (9) and (24) into $\Phi|_{\theta=0} = 0$ gives the positive equilibrium tax rate, which is represented by $T = \frac{-1}{kB'l'\overline{w}_\tau}\left(kB' - \mu\right)\Psi_t$, in this situation.[14]

Summarizing the above discussion in addition to proposition 3, the pair of $\theta = 0$ and $\Phi = 0$, which is the equilibrium condition in the model of joint provision of public inputs and also in the model of division of responsibilities, has a significantly powerful meaning, as shown by the following proposition.

Proposition 4 *The second-best rules for public expenditure of the tiers of government with the positive federal tax can be achieved irrespective of the type of federal expenditure if only the states provide public inputs by unit labour tax financing.*

3.3.2 Dual provision by states

Then, we go back to the basic model and apply another extension in which the state government provides both of public good and public input whereas the federal government provides public input only. A household's utility function is formed by $u(x,l) + b(g)$, where $b(\cdot)$ is concave and increasing in g, which is a public good provided by the state government in addition to

[14]Using kB' instead of λ, the proof of proposition 3 can be applicable to this situation.

state public input. The budget constraint of the state government slightly changes to $tl + (1-\theta)\pi + S - e - g = 0$. In this model, the second-best rule for g is represented as $\frac{b'}{u_x} = \frac{1}{1-\frac{\tau l'}{l}}$, while eqs. (3) and (4) are still needed as the second-best rules for E and e.

We again solve the Stackelberg game where the federal government at the first stage chooses T, E, S and θ and the state government at the second stage chooses t, e and g, by backward induction. The following new condition for g is derived in addition to (12) from the maximization of $v(\overline{w}) + b(g)$ by the state choosing t, e and g, subject to a new budget constraint aforementioned.

$$\frac{b'}{u_x} = \frac{1}{1 - \frac{tl'}{l} - \theta f_{ll} l'} \tag{25}$$

Eq. (25) is equivalent to (22) in Boadway and Keen (1996), and means that the second-best rule for g cannot be achieved only by $\theta = 0$, unlike the rule for state public input. Since the state government appraises the marginal cost of public good inefficiently low, state public good is overprovided.

Then, in the optimization problem of the federal government at the first stage where $k[v(\overline{w}) + b(g)]$ is maximized by choosing T, E, S and θ subject to (6) and state's reaction, the first-order conditions for the federal policy variables are consequently summarized as the follows.[15]

[15] Readers can acquire from the authors the algebraic procedure for the following equations in detail.

$$
\begin{aligned}
l\Phi + \lambda \left[\Psi_t \left(f_p p_e - 1\right) e_T + \Gamma_\tau g_T\right] &= 0, \\
-\frac{p_E}{p_e}\Phi + \lambda \left\{\Psi_t \left[\left(f_p p_e - 1\right) e_E + f_p p_E - \frac{1}{k}\right] + \Gamma_\tau g_E\right\} &= 0, \\
-\Phi + \lambda \left[\Psi_t \left(f_p p_e - 1\right) e_S + \Gamma_\tau g_S\right] &= 0, \\
\pi\Phi + \lambda \left[\Psi_t \left(f_p p_e - 1\right) e_\theta + \Gamma_\tau g_\theta\right] &= 0,
\end{aligned}
$$

where g_T, g_E, g_S and g_θ denote the changes in g in respond to the marginal changes in T, E, S and θ, derived by comparative statics using (12), (25) and the aforementioned new budget constraint of the state. In this case, the first-order conditions for T and S are not equivalent because $g_T \neq g_S l$. Ψ_t here denotes the partial derivative of an implicit function from the aforementioned state's new budget constraint, and $\Gamma_\tau = Tl'\overline{w}_\tau + \theta\pi_\tau$, while $\Phi \equiv v'\overline{w}_\tau + \lambda\left[l + (t+T)\, l'\overline{w}_\tau + \pi_\tau\right]$ again.

From these equations, we obtain that $\Phi = 0$, $\theta = 0$ and $T = 0$ are the equilibrium conditions in the case, that is;

Proposition 5 *In the model where the states provide public goods and public inputs whereas the federal government provides public input only, the federal government should set its labour tax rate to be zero in order to internalize vertical fiscal externality, and thus requires a negative inter-governmental transfer in order to finance its public input, even if the state public input is financed by unit labour tax.*

As we discussed about the neutrality theorem, the impacts of state tax and state public input on employment can completely offset each other by the optimization behaviour of the state government, and thus the vertical fiscal externalities concerning state public input provision are reduced to an increase in the federal profit tax revenue as mentioned by proposition 1. Hence, it can be internalized by choosing $\theta = 0$. However, the vertical fiscal externality concerning state public good provision in the case remains in the same ways of the BK model. Therefore, the federal government should choose zero labour tax rate to replicate the government in a unitary jurisdiction, in addition to relinquishing control of the profit tax.[16]

Proposition 5 clarifies that the vertical fiscal externality concerning state public good provision is independent from those concerning state public input provision, and that they do not cancel out each other. Therefore, we recognize that an essential problem of the vertical fiscal externality is caused by the provision of state public good not state public input in the case where a unit labour tax is employed to finance it.

4 Public finance by ad-valorem tax

Finally, we discuss a difference between the cases of unit labour tax financing and ad-valorem labour income tax financing. Although Dahlby and Wilson (2003) and Martínez (2008) have already examined the model with public input financed by ad-valorem tax, as mentioned in introduction, we re-solve the model with attention to the offset effect between vertical externalities of state tax and state public input highlighted by the neutrality theorem and proposition 1 in contrast to the Dahlby–Wilson–Martinez (DWM) model in which each of vertical fiscal externalities is assumed to be internalized respectively in the optimization problem of the federal government.

[16] One might guess the reason why the distortion concerning state public good provision is remained is that an additive-separability of utility function removes a complementarity between labour supply and public good comsumption. However we confirmed that the result does not depend on the form of the utility function, by utilizing the form $u = x + \nu(l, g)$ which has $\nu_l < 0$, $\nu_{ll} < 0$, $\nu_{gg} < 0 < \nu_g$ and $\nu_{lg} > 0$.

4.1 The model and the state's optimization

Since the model here is almost same as the basic model in section 2 except taxation, different aspects are summarized as the follows. First, household's budget constraint is described as $x = \widehat{w}l$ with a net wage rate $\widehat{w} = (1-\tau)\,w$, and then indirect utility, given by $v(\widehat{w}) = u\,(\widehat{w}l, l)$, derives an envelope property, $v' = u_x(1-\tau)l$. Second, since we obtain $w_\tau = \frac{w}{1-\tau}\frac{z}{z-\epsilon} > 0$, $\widehat{w}_\tau = (1-\tau)w_\tau - w = w\frac{\epsilon}{z-\epsilon} < 0$, and $w_p = -\frac{\epsilon}{z-\epsilon}f_{lp} > 0$ from maximization condition for the private sector,[17] eq. (2) is rewritten by using them as the following:

$$-\frac{w_p}{\widehat{w}_\tau} = \frac{f_{lp}}{w}. \tag{26}$$

Finally, the budget constraint of each tier of government is rewritten respectively as follows:

$$\begin{aligned} twl + (1-\theta)\pi + S - e &= 0 \equiv \widehat{\Psi}, & (27)\\ Twl + \theta\pi - S - \frac{E}{k} &= 0. & (28) \end{aligned}$$

Again, it is noticed that the state budget constraint is denoted by $\widehat{\Psi}$ as an implicit function for later use.

Eqs. (3) and (4) again show the second-best rules for federal public input and state public input even in the model of an ad-valorem tax financing. Then we consider the optimal policies of the federal and state governments. We again assume the game structure same as that in section 3; that is, the federal government as a Stackelberg leader chooses T, E, S, and θ at the first

[17] Each of z and ϵ respectively denotes the elasticity of the labour supply with respect to the gross wage rate and the elasticity of the demand for labour, same as in the basic model.

stage and the state governments as followers choose t and e at the second stage, taking the federal government's policy variables as given. We solve the game by backward induction.

We begin to consider the optimization problem of the follower. A representative state as a follower chooses t and e to maximize $v(\widehat{w})$ subject to (27), taking T, E, S, and θ as given. The first-order conditions for t and e are

$$\begin{aligned} v'\widehat{w}_\tau + \mu\widehat{\Psi}_t &= 0, \\ v'(1-\tau)w_p p_e + \mu\widehat{\Psi}_e &= 0. \end{aligned}$$

Combining them derives

$$\frac{\widehat{\Psi}_e}{\widehat{\Psi}_t} = (1-\tau)\frac{w_p}{\widehat{w}_\tau}p_e. \tag{29}$$

Using (1), (26) and (29), we obtain

$$\begin{aligned} f_p p_e &= 1 + \theta(f_p - f_{lp}l)p_e + T f_{lp} l p_e \\ &= 1 + (\theta f_{hp} h + T f_{lp} l)\, p_e \end{aligned} \tag{30}$$

Comparing (30) and (12), we find another term of the vertical externalities denoted by $T f_{lp} l p_e$ in addition to the original one which is represented by the increase in the federal profit tax revenue $\theta f_{hp} h p_e$.

Let us consider the meaning of this new term in the same manner to section 3, by using the following relation, which is derived from a total differential of (27) with respect to t and e under the balanced budget rule.

$$dt = -\frac{\widehat{\Psi}_e}{\widehat{\Psi}_t} de \tag{31}$$

Denoting $(l_\tau + l' w_\tau)\, dt + l' w_p p_e de$ the total impact of state tax and public input on employment and inserting (26), (29) and (31) into this impact,[18] we obtain

$$\left(l_\tau + l' w_\tau\right) dt + l' w_p p_e de = \left(\frac{\widehat{w}_\tau l'}{1-\tau}\right)\left(-(1-\tau)\frac{w_p}{\widehat{w}_\tau} p_e de\right) + l' w_p p_e de = 0. \tag{32}$$

Thus, eq. (32) gives the following.

Proposition 6 *The neutrality theorem is satisfied on the state public input financed by an ad-valorem labour income tax.*

It is significant in the model that the state public input is neutral to employment irrespective of the type of labour taxation; a unit labour tax or an ad-valorem labour income tax. This means that we do not need to be concerned about the elasticities of labour demand and supply in the model of state public input provision in contrast to Kotsogiannis and Martínez (2008) who compare the effect of the shrinkage in employment and that of a rise in the gross wage rate in order to clarify the sign of vertical externality.

Then, denoting by $Tr_\tau dt + Tr_p p_e de$ the sum of the impacts of state tax and state public input on the federal labour income tax revenue with defining a gross labour income $r = wl$, we obtain by using (26), (29), (31) and (32) the following:

[18] In the case of ad-valorem tax, we derive $l'(=\frac{\partial l}{\partial w})$ and $l_\tau(=\frac{\partial l}{\partial \tau})$ from comparative statics on the first-order conditions of houshold's maximization. Futhermore, $l_\tau = -\frac{w}{1-\tau} l'$ is derived from the assumption on the utility function mentioned by footnote 4.

$$
\begin{aligned}
Tr_\tau dt + Tr_p p_e de &= Tw\left[\left(l_\tau + l' w_\tau\right) dt + l' w_p p_e de\right] + Tl\left[w_\tau dt + w_p p_e de\right] \\
&= T f_{lp} l p_e de. \qquad (33)
\end{aligned}
$$

Eqs. (32) and (33) give a following consideration about (30).

Proposition 7 *In the case of the state public input financed by an ad-valorem labour income tax, the vertical fiscal externalities are reduced to the increase in the federal profit tax revenue and the change in the federal labour income tax revenue with respect only of the rise in the gross wage rate.*

In the case of ad-valorem tax-financed public input, a rise in the wage rate not only causes the profit gain but also directly affects the federal labour tax revenue, while the level of employment is consequently unchanged because the shifts of the labour demand and supply curves cancel each other out.[19] It is remarkable that proposition 7 does not depend on the extent of the elasticity of labour demand and supply. However, remembering that the signs of $\theta f_{hp} h$ and $f_{lp} l$ in (30) are defined as positive, we recognize that the sign of the reduced vertical fiscal externality is considered to be each of following three cases: (i) consistently positive as $T > 0$, (ii) zero when $T = \theta = 0$, or (iii) depending on the size of the absolute values of $\theta f_{hp} h$ and $T f_{lp} l$ as $T < 0$.

4.2 The federal's optimization

Next, we consider the maximization problem of the federal government in a similar way of subsection 3.2. As aforementioned, since it is obvious that T should be zero if θ is exogenously become zero, we again assume that the

[19] The evidence was not remarked by Dahlby and Wilson (2003) and Martínez (2008).

federal government endogenously chooses T and θ in addition to E and S to maximize $kv(\widehat{w})$ subject to (28) and the reaction of the state. By similar procedures for comparative statics with those in subsection 3.2, we summarize the following first-order conditions for the federal policy variables, respectively:

$$\begin{aligned}
\widehat{\Delta}_\tau (1 + t_T) + \widehat{\Delta}_p p_e e_T + \lambda wl &= 0, & (34)\\
\widehat{\Delta}_\tau t_E + \widehat{\Delta}_p (p_e e_E + p_E) - \frac{\lambda}{k} &= 0, & (35)\\
\widehat{\Delta}_\tau t_S + \widehat{\Delta}_p p_e e_S - \lambda &= 0, & (36)\\
\widehat{\Delta}_\tau t_\theta + \widehat{\Delta}_p p_e e_\theta + \lambda \pi &= 0, & (37)
\end{aligned}$$

where $\widehat{\Delta}_\tau = v_w \widehat{w}_\tau + \lambda (Tr_\tau + \theta \pi_\tau)$ and $\widehat{\Delta}_p = v_w (1 - \tau) w_p + \lambda (Tr_p + \theta \pi_p)$.

From the results of the comparative statics, we derive $1 + t_T = -t_S wl$ and $e_T = -e_S wl$. Therefore, we obtain the following remark.

Remark 2 *S is also incompatible policy variable with T in the model where both tiers of government provide public inputs financed by ad-valorem labour income taxes.*

As explanation for remark 1, the federal government can ignore the distortion via the reaction of the state in a labour market also in the case of the ad-valorem labour income tax due to the neutrality theorem confirmed by proposition 6. Therefore, the negative inter-governmental transfer is not needed to avoid the distortion and finance the federal public input also in this model.

Again, we can reduce (34) – (37) to the following equations after algebraic manipulations:

$$wl\widehat{\Phi}+\lambda\widehat{\Psi}_t\left(f_p p_e-1\right)e_T=0, \quad (38)$$

$$-\frac{p_E}{p_e}\left(1-\tau\right)\widehat{\Phi}+\lambda\widehat{\Psi}_t\left[\left(f_p p_e-1\right)e_E+f_p p_E-\frac{1}{k}-\left(t+T\right)\frac{p_E}{p_e}\right]=0, \quad (39)$$

$$\pi\widehat{\Phi}+\lambda\widehat{\Psi}_t\left(f_p p_e-1\right)e_\theta=0, \quad (40)$$

where $\widehat{\Phi}\equiv v'\widehat{w}_\tau+\lambda\left[wl+\left(t+T\right)r_\tau+\pi_\tau\right]$ and $\widehat{\Psi}_t\equiv wl+tr_\tau+\left(1-\theta\right)\pi_\tau$.

In the same way of (21) – (23), eqs. (38) and (40) derive that $\widehat{\Phi}=0$ and $f_p p_e-1=0$, and thus $\theta f_{hp}h+Tf_{lp}l=0$ from (30). In addition to them, (39) shows that $t+T=0$ is the another equilibrium condition for the current model. Since these equilibrium conditions, $\theta f_{hp}h+Tf_{lp}l=0$ and $t+T=0$, derive $t=\frac{f_{hp}h}{f_{lp}l}\theta$, we conclude that the federal labour income tax rate T is negative at the equilibrium while t is positive. Therefore, we obtain the following proposition.

Proposition 7 *In the model where both tiers of government jointly provide public inputs by ad-valorem labour income tax financing, the federal government should set its labour income tax rate to be negative in order to internalize positive vertical fiscal externality which is denoted by an increase in the federal profit tax revenue. However, in this case, the federal government can replicate the government in a unitary jurisdiction without a negative inter-governmental transfer.*

As we have explained in subsection 3.1, the state treats a partially shifting of the profit gain to the federal revenue as the MCPF, and thus under-provides its public input. In order to correct such a decision-making of the state and let its provision of state public input to achieve a second-best level, the federal government sets a subsidy for labour income as a device to offset such a partially shifting of the profit gain at a margin. In this situation, the federal government can efficiently provide its public input which is financed by the federal profit tax, when the absolute value of the subsidy rate T is chosen to be equal to the state labour income tax rate t.

It is noticed that the condition $\theta f_{hp}h + T f_{lp}l = 0$ is applied only to the margin not to the level, and thus does not mean that the federal profit tax revenue $\theta\pi$ is equivalent to the amount of labour income subsidy Twl ($T < 0$). That is, in the equilibrium, the federal government finances its public input and the labour income subsidy by the profit tax revenue without a negative inter-governmental transfers from the state governments.

4.3 Comparison with the related literature

The results in this section can be compared with those in Dahlby and Wilson (2003) and Martínez (2008). First, eq. (30) is corresponding to (19) in Dahlby and Wilson (2003). However, since they implicitly assume a positive federal labour income tax rate and discuss only on the optimal condition for the state policy without solving the maximization problem of the federal government, they suggest only that a matching grant as an additional federal policy instrument is needed to internalize vertical fiscal externality. Therefore, their remarks mean that the federal government cannot achieve an efficient provision of public inputs by existing policy instrument in the model of ad-valorem tax financing.

On the other hand, our analysis shows that the federal government can achieve the second-best allocation by choosing a labour income subsidy and, however, does not need the additional instrument at the equilibrium of the Stackelberg game which is a general procedure in the literature on vertical fiscal externalities.

Next, from our consideration, it seems to be tricky to impose the condition $R_t = R_e = 0$ on solving the maximization problem of the federal government in Martínez (2008),which means that each of vertical externality of state tax and state public input is respectively internalized. To show that, we derive the following two equations by partial differentials of (28) with respect to t and e:

$$\begin{aligned} Tr_\tau + \theta\pi_\tau &= 0, \\ (Tr_p + \theta\pi_p)\, p_e &= 0. \end{aligned}$$

Combining them, we obtain

$$T\left[(1+\epsilon)f_p - \epsilon f_{lp} l\right] = 0, \tag{41}$$

where ϵ denotes the elasticity of the demand for labour, which we have used in the paper.

In order to obtain the positive tax rate of the federal labour income tax resulted in Martínez (2008), the term in the square brackets on the LHS of (41) must be zero; that is,

$$\frac{f_{lp} l}{f_p} = 1 + \frac{1}{\epsilon}. \tag{42}$$

The LHS of (42) is same as the elasticity of marginal productivity of state public input with respect to employment mentioned in Martínez (2008).

Although Martínez (2008) shows the possibility of the positive tax rate as this elasticity exceeds 1 under a specific production technology by solving the optimization problem of the federal government under the condition

$R_t = R_e = 0$, eq. (42) indicates an inconsistency in the procedure because ϵ is defined to be negative.[20]

On the other hand, our findings show that such a specific condition is not needed to solve the optimization problem and obtain a reasonable result for a policy instrument choice of the federal government.

5 Concluding remarks

We investigated the role of labour (income) tax-financed public inputs by different tiers of government in a model of vertical fiscal externalities. First, we studied the model of joint provision of public inputs financed by unit labour taxation, which has a characteristic of counter-setting to the Boadway–Keen model with respect to the type of public expenditure. Then, we extended the model to the ad-valorem labour income taxation, which is the joint provision of public inputs version of the Dahlby–Wilson–Martínez model.

Our findings are summarized as the follows. First, the state public input is neutral to employment irrespective of the type of taxation; a unit labour tax or an ad-valorem labour income tax. Secondly, the neutrality theorem derives that the vertical fiscal externalities are reduced to the increase in the federal profit tax revenue in the case of a unit tax, and the change in the federal labour income tax revenue in addition to the profit tax revenue increase in the case of an ad-valorem tax. Thus, thirdly, the federal government can internalize vertical fiscal externalities and replicate the government in a unitary jurisdiction by relinquishing control of the profit tax even if it chooses a positive tax rate in the case of a unit tax. Furthermore, fourthly, the second-best rules for public expenditure of the tiers of government with the

[20] Notice that eq. (42) can be applied to the discussion in Martínez (2008) because it does not depend on the property of production function.

positive federal tax can be achieved irrespective of the type of federal expenditure if only the states provide public inputs by unit labour tax financing. On the other hand, finally, the federal government chooses a labour income subsidy (the negative tax rate of its labour income tax) in order to internalize positive vertical fiscal externality which is denoted by the increase in the profit tax revenue, however, it can replicate the government in a unitary jurisdiction without a negative inter-governmental transfer in the case of an ad-valorem tax.

Our findings give contrasting implications to those offered by Boadway and Keen (1996). That is, the federal government can choose a positive labour tax rate under the second-best allocation in the model of vertical fiscal externalities with public input provision by unit labour tax financing. In addition, the inter-governmental transfer is not necessary for the achievement.

The federal subsidy for labour income derived from our result can be considered as the reasonable embodiment of the implication suggested by Dahlby and Wilson (2003). Moreover, our conclusion is distinctive in that it does not depend on the condition that requires an additional matching grant to be needed or a specific procedure for solving the optimization problem; this is in contrast to the remarks by Dahlby and Wilson (2003) and Martínez (2008).

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