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Abstract

In the two-country asymmetric tax competition model described in Wilson (1991) and Bucovetsky (1991), it is impossible for both countries to harmonize their tax rates to improve welfare. However, in a multilateral asymmetric tax competition model, we show that partial tax coordination improves the welfare of all countries irrespective of existing inside or outside the coalition. In addition, we deduce that this coalition is stable under particular conditions.

JEL classification: H70, H77

Keywords: Asymmetric tax competition, Tax coordination

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1 Introduction

This paper aims to investigate the possibility of partial tax coordination in multilateral asymmetric tax competition. The seminal literature of Wilson (1991) and Bucovetsky (1991), which illustrated asymmetric tax competition in a two-country model, suggest the impossibility of tax harmonization between both countries due to an objection from small country whose welfare is worsened by the harmonization. Furthermore, in the case of consumption tax competition between two asymmetric countries, Kanbur and Keen (1993) indicated that '*the smaller country loses from harmonization to any tax rate between those set in the non-cooperative equilibrium*' (p. 877). From these negative observations against tax harmonization in two-country asymmetric tax competition models, we consider a simple question: does tax harmonization also fail to overcome the inefficiency from tax competition in a *multilateral* asymmetric world?

Over the past two decades, many studies have been conducted on tax competition[1], and it has been proposed that the harmful effects from tax competition must be eliminated in a real world[2]. Cnossen (2003) shows that in the European Union (EU), the average rates of capital tax (23%) and consumption tax (25%) are inefficiently lower than that of labour tax (37%) and suggests the influence of tax competition.

In this literature, tax harmonization has been considered as a representative procedure to overcome the inefficiency from tax competition. The famous comments by Keen and Marchand (1997) clearly show that countries engaged in capital tax competition are better off when they raise their tax rates jointly. Even though almost all literature that mention the benefit of tax harmonization have dealt with it in a symmetric situation, the real world is asymmetric. Based on Cnossen's observation, particularly about capital taxation, it is apparent that the EU member states have a 'crazy quilt' of taxes on capital income with 'widely diverging' effective tax rates; for example, Ireland's effective rate of corporation tax is 10 percent while that of Germany is 41 percent (Cnossen 2003, p. 637).

In the context of asymmetric tax competition, it is well-known that tax harmonization has worsened the welfare of small countries, and thus it is opposed. From this suggestion, it seems that unanimously accepted full tax harmonization does not exist in asymmetric tax

[1] See. Wilson (1999) and Zodrow (2003).

[2] See. European Commission (2001) and OECD (2000).

competition. On the other hand, Konrad and Schjelderup (1999) show that partial tax coordination among numerous but symmetric countries can improve the welfare of all countries, if the behaviours of the countries engaged in tax competition are strategic complements, using the procedure proposed by Deneckere and Davidson (1985).

Two issues should be considered in the context of tax harmonization. The first issue is a coalition formation problem that relates to the coalition stability analyzed by Burbidge et al. (1997). However, it does not seem to be a very serious problem if we imagine the situation in which countries are linked via a common institution, such as members of the EU or regions in a federal country. The second issue is whether a group of countries can be better off under tax coordination. It is unacceptable for the smaller country in a two-country model which has no member of the group to raise tax rate jointly at the non-cooperative Nash equilibrium, because capital outflow will worsen the welfare of the smaller country. The most distinctive point of a multilateral countries model from a two-country model is that a small country can collude with other small countries. Similar to the observation of Konrad and Schjelderup (1999), it is also possible that the fiscal externality effect existing among members of a coalition group is internalized by partial tax coordination within the group. Thus, we can expect that small countries can be better off by raising their tax rates jointly in multilateral asymmetric tax competition.

Some helpful comments were proposed by Peralta and van Ypersele (2005, 2006), who also investigate asymmetric capital tax competition among more than two countries. Their research is different from this resesrch in that they did not consider the co-operative behaviour of countries in a coalition group but only treated introducing alternative tax schemes such as a minimum tax rate or a certain tax range.

This paper comprises the following sections. We outline the multilateral asymmetric tax competition model in section 2 and characterize the non-cooperative equilibrium in section 3. Subsequently, we analyze the benefit of partial tax coordination in section 4. In addition, we verify the potential of models such as partial tax coordination by a numerical examination in a specified model in section 5. Finally, we provide the concluding remarks in section 6.

2 Basic Model

This section describes the asymmetric tax competition model for source based capital taxation. There are asymmetric $M + N$ countries in the economy. We define that country $m(= 1, ..., M)$ is a large country that belongs in the set $(\mathfrak{M})$ of large countries and that country $n(= M + 1, ..., M + N)$ is a small country that belongs in the set $(\mathfrak{N})$ of small countries. A large country differs from a small country only with respect to the population size.

Production and Factor endowments

In each country, two production factors, capital K and labour L, are used in the production of single private goods. We describe a linear homogeneous production function $F(K, L)$ as $f(k)$ with a capital-labour ratio $k = K/L$, $f'(k) > 0$ and $f''(k) < 0$. Assuming that firms behave competitively, the gross rent for production factors is described as follows: $f'(k) = \rho + t$, where t is capital tax rate, and ρ is the net return of capital. $\varpi = f(k) - kf'(k)$, where ϖ is the wage.

We assume that all households in the economy have endowments of both production factors. Factor endowment per capita, which is described as $\overline{k}(= \overline{K}/\overline{L})$, is equivalent in all countries, that is, $\overline{k}_m = \overline{k}_n = \overline{k}_W$, where $\overline{k}_W$ is the average level of capital endowment in the whole economy. Since households in each country are homogeneous and inelastically supply one unit of labour, $\overline{L}$ represents the number of households in a country. We define that $s_m = \dfrac{\overline{L}_m}{\sum_h \overline{L}_h}$ is the proportion of the large country population and $s_n = \dfrac{\overline{L}_n}{\sum_h \overline{L}_h}$ is the proportion of the small country population, where $h = m, n$, hence $\sum_m s_m + \sum_n s_n = 1$. We assume that $s_m > s_n$.

Capital market

Households can choose to invest their capital where they can obtain the highest net rent in the world capital market. This implies that the following arbitrage condition holds in equilibrium:

$$f'(k_m) - t_m = f'(k_n) - t_n = \rho \quad \forall m \text{ and } n. \tag{1}$$

The equilibrium of a capital market is defined by the following equation:

$$\sum\nolimits_m s_m k_m + \sum\nolimits_n s_n k_n = \overline{k}_W. \tag{2}$$

Considering the competitive behaviour of the firm and the equilibrium condition of the capital market, we obtain the following marginal effect of capital tax rate on the net return of capital and the amount of capital investment in country h.

$$\frac{\partial \rho}{\partial t_h} = \rho_{t_h} = -\frac{s_h k'_h}{\sum_h s_h k'_h}, \tag{3}$$

$$\frac{\partial k_h}{\partial t_h} = k'_h \frac{\sum_{-h} s_{-h} k'_{-h}}{\sum_h s_h k'_h} = k'_h(1+\rho_{t_h}), \tag{4}$$

$$\frac{\partial k_h}{\partial t_{-h}} = -k'_h \frac{s_{-h} k'_{-h}}{\sum_h s_h k'_h} = k'_h \rho_{t_h}, \tag{5}$$

where $k'_h = \dfrac{1}{f''(k_h)}$, $h = n, m$.

From the definition of the population proportion, it is obvious that $-1 < \rho_{t_h} < 0$, hence $\dfrac{\partial k_h}{\partial t_h} < 0$, $\dfrac{\partial k_h}{\partial t_{-h}} > 0$.

Household

Homogenous households spend their income from labour supply and capital investment for consumption of private goods (x) and public goods (g). The utility function of a representative household is defined as a quasi-concave function, that is, $u = u(x, g)$, $u' > 0$, $u'' < 0$. The budget constraint of a household is $x = f(k) - tk - \rho(k - \overline{k})$.

Government

Each government provides public goods. The marginal rate of transformation between private and public goods is constantly unity. The provision of public goods is financed by the capital tax, that is, $g = tk \quad (t > 0)$.

3 Non-cooperative Equilibrium

Each government chooses the capital tax rate in order to maximize the utility of households in the country. The maximization problem of a government is

$$\max_t \; u(x, g) \tag{6}$$

$$\begin{aligned} s.t. \; x &= f(k) - tk - \rho(k - \overline{k}) \\ g &= tk. \end{aligned}$$

From the first-order condition, we obtain the optimal condition of the capital tax rate.

$$\frac{u_g}{u_x} = \frac{1+\rho_{t_h}(1-\overline{k}/k_h)}{1+\epsilon_{kt}} \quad, h = m, n \quad and \quad \epsilon_{kt} = \frac{t_h}{k_h} k_h' \left(1+\rho_{t_h}\right). \tag{7}$$

Here ϵ_{kt} is a negative tax elasticity of demand for capital, $\epsilon_{kt} = \dfrac{t_h}{k_h}\dfrac{\partial k_h}{\partial t_h}$.

We realize that various situations could occur from the above optimal condition in a generalized model; for example, a negative effect on the amount of public goods with a marginal rise in tax rate, over-provision (or under-provision) of public goods. However, in our paper, we focus only on the difference between implications in a two-country model and a multilateral countries model. For this purpose, we set the following assumptions that clarify the situation in which public goods are under-provided. They were mentioned in Wilson (1991).

Assumption 1

(a) $-1 < \epsilon_{kt} < 0 \quad \forall t_h$.

(b) $\left|\rho_{t_h}\left(1-\overline{k}/k_h\right)\right| < |\epsilon_{kt}| \quad \forall$ *capital importers.*

Assumption 1(a), which is usually employed in the literature of tax competition, guarantees that $\frac{dg_h}{dt_h} > 0 \ \ \forall t_h \ (t_h > 0;\ h = m, n)$. Assumption 1(b) makes the numerator on the RHS of equation (7) larger than the denominator on the RHS in the case of capital importers and guarantees that public goods are under-provided in capital importing countries.

Moreover, we introduce some assumptions on the reaction function in a similar manner as adopted in Konrad and Schjelderup (1999) for meaningful interpretations about later analyses. Let $\mathbf{t} \equiv (t_1, t_2, ..., t_{M+N})$ be a vector of tax rates and $\mathbf{t}_{-h} \equiv (t_1, t_2, ..., t_{h-1}, t_{h+1}, ..., t_{M+N})$ be a vector of tax rates except country h (where $h = m, n$). They are chosen from the set $\mathcal{T}$ of all feasible tax vectors. Let $\Theta_h(\mathbf{t}_{-h}) = \left\{\arg\max_{t_h \in \mathcal{T}} \ u_h\left(t_h; \mathbf{t}_{-h}\right)\right\}$ be the reaction correspondence of country h.

Assumption 2 (Konrad and Schjelderup 1999)

(a) *The reaction correspondence is singleton-valued on set $\mathcal{T}$. It can be represented by a well-defined reaction function $\beta_h(\mathbf{t}_{-h})$, which maps elements of $\mathcal{T}$ to the country's optimal tax rate.*

(b) *The reaction function $\beta_h(\mathbf{t}_{-h})$ is continuously differentiable and globally non-decreasing* *in each component of* $\mathbf{t}_{-h}$*. That is,* $\dfrac{\partial \beta_h(\mathbf{t}_{-h})}{\partial t_{-h}} \geq 0$ *for all* h *for all* $\mathbf{t}_{-h}$*. Moreover, the slope of the reaction function at the equilibrium satisfies* $\sum_{-h}\left(\dfrac{\partial \beta_h}{\partial t_{-h}}\right) < 1$.

(c) *If countries choose their tax rates non-cooperatively, the unique Nash equilibrium tax vector* $\mathbf{t}^*$ *exists.*

These assumptions imply that the behaviours of countries are strategic complements, hence the reaction functions are upward sloping, and that the asymmetric tax competition model in our paper has a unique Nash equilibrium. Of course, we notice that some technical specifications about preferences and production technologies are needed to allow the reaction functions to satisfy the aforementioned assumptions[3]. However, since our purpose does not include acquiring the existence of Nash equilibrium in our model, we apply these assumptions without proof of the existence of equilibrium, similar to the literature on other tax competition.

With these assumptions, we derive the relationship between the features of the reaction function and the proportion of the country population as follows.

Lemma 1 *Under assumptions 1 and 2, reaction functions are increasing in the proportion of the country population, that is,* $\dfrac{\partial \beta_h(s_h; \mathbf{t}_{-h})}{\partial s_h} > 0$ *for all* s_h *and for all* $\mathbf{t}_{-h}$*.*

Proof. Suppose a symmetric world in which all countries have the same population size ($s_n = s_m$). In this situation, the optimal tax rate of country h is derived by the following condition:

$$\frac{u_g}{u_x} = \frac{k_h}{k_h + t_h k_h' \left(1 + \rho_{t_h}\right)}. \tag{8}$$

Then we assume an exogenous increase in the proportion of the country h's population (s_h), while the tax rates (t_{-h}) and the population proportions (s_{-h}) of all other countries are held to be fixed. The LHS of equation (8) is not affected by a rise in s_h, since the preference about a consumption pattern between private and public goods does not change as the population size changes. On the other hand, on the basis of equations (3) and (4), this exogenous increase in s_h raises $\left|\rho_{t_h}\right|$ and reduces the degree of $\frac{\partial k_h}{\partial t_h}$ holding t_h to be fixed. This means that a large country has great power in a capital market and can keep capital

[3] See, for example, Laussel and Breton (1998) characterizing the set of Nash equilibria under specific assumptions for fiscal competition.

investment in the country by assigning the tax burden from the cost of capital demand to the net return of capital[4]. Thus, the RHS of equation (8) is reduced due to an increase in its denominator caused by the rise in $|\rho_{t_h}|$. Therefore, t_h must rise to maintain the equality of equation (8).

Since it is assumed that the reaction function is continuous and globally non-decreasing in each component of $\mathbf{t}_{-h}$, a different case in which the tax rate of other countries is the alternative level such as t'_{-h} will be handled similarly. Therefore, $\dfrac{\partial \beta_h(s_h; \mathbf{t}_{-h})}{\partial s_h} > 0$ for all $\mathbf{t}_{-h}$. ■

Considering this effect of population proportion on the reaction function, $s_m > s_n$ leads $\beta_m > \beta_n$. Thus, we obtain the following proposition about the relationship between the proportion of the country's population and the equilibrium tax rate under the assumptions allowing the existence of Nash equilibrium.

Proposition 1 *If the reaction functions are characterized by assumption 2, the equilibrium tax rate of the large country (t_m^*) exceeds that of the small country (t_n^*).*

With equations (1) and (2), and the equilibrium conditions of a capital market, proposition 1 immediately implies the following corollary.

Corollary 1 *If the reaction functions are characterized by assumption 2, the capital invested in the large country (k_m^*) is less than that in the small country (k_n^*) under the non-cooperative Nash equilibrium. This implies $k_m^* < \overline{k} < k_n^*$.*

Then we consider the utility difference that is related to the difference in equilibrium tax rates. The proposition about the utility difference needs the following lemma, which corresponds to lemma 1 in Wilson (1991: p. 430).

Lemma 2 *Given any initial values of t_n and t_m, a reduction in t_m lowers country n's public goods supply, irrespective of size differences among countries. If $t_m > (<) t_n$ before and after the tax change with strict inequality before, then country n's private goods consumption also falls (rises).*

[4] Hoyt (1991) clearly indicated that a more monopolistic country can impose a heavier burden of capital tax on an owner of capital and prevent capital investment from flowing outside the country.

Proof. Let us suppose two countries m and n. On the basis of equation (3), a reduction in t_m raises ρ. This rise in ρ increases the payment for capital demand in country n, with t_n held to be fixed. This causes the capital outflow from country n and a decrease in $g_n(= t_n k_n)$. Then let $t_m > t_n$. From the budget constraint of a household, the marginal change of private consumption in country n is

$$\frac{\partial x_n}{\partial t_m} = -\left(k_n - \overline{k}\right)\rho_{t_m}. \tag{9}$$

$t_m > t_n$ implies $k_m < \overline{k} < k_n$, similarly we obtain the induction of corollary 1. Thus, the sign of equation (9) is positive; that is, a reduction in t_m also reduces country n's private goods consumption. In the case where $t_m < t_n$, the sign of equation (9) is negative due to an inverse order of the amount of capital; $k_n < \overline{k} < k_m$. Therefore, a reduction in t_m increases the private goods consumption in country n. ■

Using lemma 2, we obtain the proposition about the relationship between the utility difference and the equilibrium tax rates.

Proposition 2 *If the reaction functions are characterized by assumption 2, the utility of the large country (u_m^*) is less than that of the small country (u_n^*) under the non-cooperative Nash equilibrium ($t_m^* > t_n^*$).*

Proof. Similar to Wilson (1991), we can use a revealed preference argument that describes that the equilibrium consumption bundle of the large country (x_m^*, g_m^*) exists in the interior of the small country's consumption possibility frontier (CPF: $a_n b_n$ in Fig. 1) in multilateral asymmetric tax competition. The discussion is divided in two parts. First, we show that the small country's CPF lies above the production possibility frontier (PPF: ab in Fig. 1) of the whole economy. Next we describe that the large country's CPF lies below the PPF as the second part of the proof. We note that the PPF whose slope is –1 represents the CPF in a situation in which all countries choose the same tax rate.

The CPF of the small country represents the feasible set of consumption bundles under the tax rates of all large countries to be fixed as t_m^*. If all small countries choose t_m^*, the consumption bundle of the small country n $(\overline{x}, \overline{g})$ is located on the PPF. At the point $(\overline{x}, \overline{g})$, the slope of the CPF of country n is steeper than that of the PPF because it is shown as

$-\dfrac{\overline{k}}{\overline{k}+t_m^* k_n'(1+\rho_{t_n})}$, taking the tax rates of all other countries as given[5]. Since proposition 1 shows that the equilibrium tax rate of the small country is less than that of large country, any feasible tax rates, such as $\widetilde{t}_n$, are less than t_m^* as the large countries choose t_m^*.

Let us now suppose that all large countries reduce their tax rates in order to harmonize. At any point $(\widetilde{x}_n, \widetilde{g}_n)$ corresponding to $\widetilde{t}_n$, lemma 2 tells us that such exogenous reduction in the tax rates of large countries (t_m^*) lowers both public and private goods consumption in country n. Therefore, we recognize that the bundle $(\widetilde{x}_n, \widetilde{g}_n)$ locates on the upper-left of $(\overline{x}, \overline{g})$ along the CPF, which lies above the PPF as denoted by z in Figure 1.

Next, let us return to the situation in which all countries choose t_m^*. Since the amount of capital investment in the country is equivalent to the capital endowment in this situation, on the basis of lemma 2, country m's private goods consumption does not change while public goods supply is lowered as all small countries reduce their tax rates jointly[6]. This implies that the equilibrium consumption bundle of the large country (x_m^*, g_m^*) exists horizontally left of $(\overline{x}, \overline{g})$. The slope of the CPF of the large country at (x_m^*, g_m^*) is shown as $-\dfrac{k_m^*+\rho_{t_m}(k_m^*-\overline{k})}{k_m^*+t_m^* k_m'(1+\rho_{t_m})}$. Then we assume that the large country m reduces its tax rate to t_n^*, which brings country m a corresponding consumption bundle $(\underline{x}, \underline{g})$, taking the tax rates of all other countries as given. The slope of the CPF changes from $-\dfrac{k_m^*+\rho_{t_m}(k_m^*-\overline{k})}{k_m^*+t_m^* k_m'(1+\rho_{t_m})}$ to $-\dfrac{\overline{k}}{\overline{k}+t_n^* k_m'(1+\rho_{t_m})}$; that is, the slope of the CPF still is steeper than that of the PPF while it moderates. This implies that the CPF of the large country ($a_m b_m$ in Fig. 1) lies below the PPF in the under-right area from $(\underline{x}, \underline{g})$. Therefore, it is shown that the equilibrium consumption bundle of the large country (x_m^*, g_m^*) exists in the interior of the small country's CPF. From the above discussion, it is apparent that $u_m^*(x_m^*, g_m^*) < u_n^*(x_n^*, g_n^*)$. ■

4 Partial Coordination

In the two-country asymmetric tax competition model described in Wilson (1991) and Bucovetsky (1991), it is impossible that both countries harmonize their tax rates to the same level because there is no incentive to harmonize. Thus, it is considered that the subsidy

[5] In the case where all countries choose the same tax rate, the amounts of capital invested in countries are equivalent; that is, $k_n = k_m = \overline{k}$.

[6] The amount of capital investment in country m is also reduced by the reduction in the tax rates of small countries. However, the marginal reduction in the large country's production is offset by the marginal reduction in the cost for capital demand: $[f'(k_m) - (t_m + \rho)] \frac{\partial k_m}{\partial t_n} = 0$. Thus, private goods consumption in country m does not change without the effect of the change in the capital market price through capital trade.

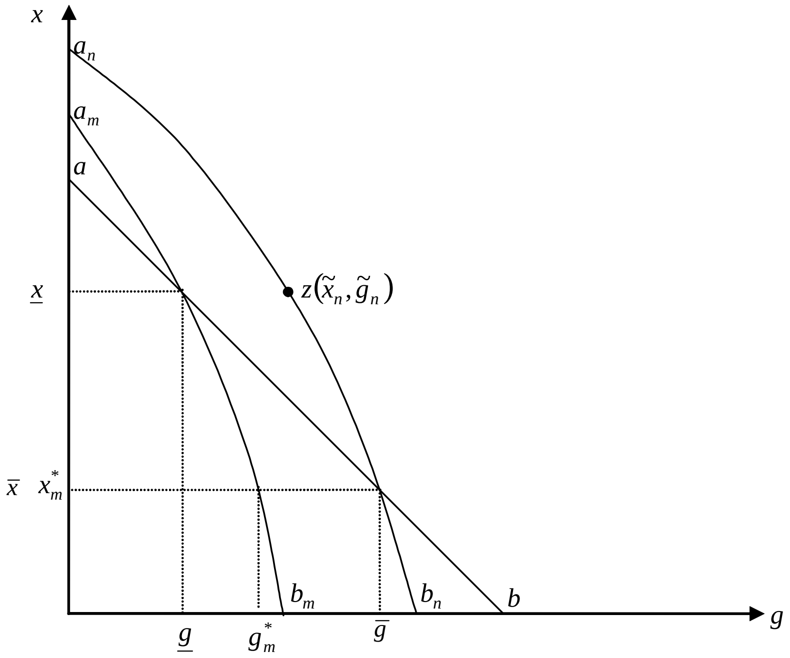

Figure 1: Proof of proposition 3

scheme proposed by DePater and Myers (1994) is the only alternative to intervention by the central planner to improve welfare in asymmetric tax competition. Recently, Peralta and van Ypersele (2006) suggested that a tax range scheme among asymmetric countries engaged in tax competition may improve the welfare level. Their implication shows another possibility. However, tax coordination has not been mentioned. Therefore, we investigate the possibility of tax coordination in asymmetric tax competition.

Since coalition stability has been doubted on the basis of the comments by Burbidge et al. (1997), who concluded that equilibrium with full coordination is generally not unique in asymmetric situations, we consider partial tax coordination among same-size countries to avoid the problem of coalition stability. It seems that the possibility of deviation is relatively low in a coalition in which the members are identical if the members are sufficiently better off by colluding.

We consider two types of partial tax coordination: only among small countries and only among large countries. Our interest is 'which tax coordination can improve a member's welfare?' To answer this question, we employ a framework of analysis used by Konrad and Schjelderup (1998).

4.1 Small countries' coordination

Let us consider that all N small countries collude by co-operatively choosing a common tax rate, but non-cooperatively react to a large country outside the coalition. In addition, each of the large countries adjusts its tax rate to the common tax rate decided by the coalition of small countries non-cooperatively.

Let $\mathfrak{N}$ be the set of small countries in the coalition. The members of the coalition choose a common tax rate ($t_{\mathfrak{N}}$) in order to maximize a joint utility of the members. Large countries simultaneously choose their tax rates, taking $t_{\mathfrak{N}}$ as given. Now $t_{\mathfrak{N}}$ is defined as $t_{\mathfrak{N}} \equiv t_{M+1} = , ..., = t_{M+N}$. In the following equation, t_i and t_m represent tax rates of the other small country and large country, respectively.

In this situation, the utility of small country n changes as follows with the marginal rise in a common tax rate.

$$\begin{aligned} \frac{du_n}{dt_{\mathfrak{N}}} &= (u_g - u_x)\, k_n + u_g t_n \left(\frac{\partial k_n}{\partial t_n} + \sum_{i \in \mathfrak{N}} \frac{\partial k_n}{\partial t_i} + \sum_m \frac{\partial k_n}{\partial t_m} \frac{\partial t_m}{\partial t_{\mathfrak{N}}} \right) \\ &\quad - u_x \left(k_n - \overline{k} \right) \left(\frac{\partial \rho}{\partial t_{\mathfrak{N}}} + \sum_m \frac{\partial \rho}{\partial t_m} \frac{\partial t_m}{\partial t_{\mathfrak{N}}} \right), i(\neq n) \in \mathfrak{N}. \end{aligned} \tag{10}$$

$\frac{\partial t_m}{\partial t_{\mathfrak{N}}} \left(\equiv \sum_{i \in \mathfrak{N}} \frac{\partial t_m}{\partial t_n} \right)$ is the large country's reaction to the tax change in small countries manipulating a common tax rate, and $\frac{\partial \rho}{\partial t_{\mathfrak{N}}} \left(\equiv \sum_{n \in \mathfrak{N}} \frac{\partial \rho}{\partial t_n} \right)$ is a total effect of the tax change in small countries on the net return of capital.

Equation (10) shows three well-known effects in asymmetric tax competition. The first effect provided by the first term on the RHS of equation (10) is the direct tax effect with the amount of capital investment in the small country held to be fixed as the non-cooperative equilibrium level.

The second effect provided by the second term is the fiscal externality effect. In a non-cooperative Nash game, this effect is represented only by the first part in a bracket of the second term. This means that a rise in one's own tax rate causes capital outflow. On the other hand, in such a coalition, a part of fiscal externality is internalized by the corporative rise in the tax rate of the other coalition members. Moreover, the capital inflow caused by the reaction of outsiders weakens the fiscal externality effect. However, since the impact of own one's tax rate is absolutely larger than the sum of the latter two parts, the fiscal externality is not completely internalized.

The third effect is the trade effect that is represented by the third term on the RHS of equation (10). DePater and Myers (1994) called this effect 'a pecuniary externality'. Since the small country is a capital importer, this term shows that private goods consumption is increased by the reduction in the payment for capital.

Evaluating equation (10) with the optimal condition in equation (7) at the non-cooperative Nash equilibrium, and using assumption 2 about the feature of the reaction function, we obtain

$$\frac{du_n}{dt_{\mathfrak{N}}} = \left[-u_x\left(k_n^* - \overline{k}\right) + u_g t_n^* k_n'\right]\left(\sum_{i\in\mathfrak{N}}\frac{\partial\rho^*}{\partial t_i} + \sum_m \frac{\partial\rho^*}{\partial t_m}\frac{\partial t_m}{\partial t_{\mathfrak{N}}}\right) > 0. \tag{11}$$

In equation (11), the first part on the RHS, denoted by $\left[-u_x\left(k_n^* - \overline{k}\right)\right]\left(\sum_{i\in\mathfrak{N}}\frac{\partial\rho^*}{\partial t_i} + \sum_m \frac{\partial\rho^*}{\partial t_m}\frac{\partial t_m}{\partial t_{\mathfrak{N}}}\right)$, means an improvement of utility through an increase in the private goods consumption due to a reduction in the payment for capital.

Since the fiscal externality of one's own tax is offset by the effect of the rise in its tax rate, the second part, denoted by $\left[u_g t_n^* k_n'\right]\left(\sum_{i\in\mathfrak{N}}\frac{\partial\rho^*}{\partial t_i} + \sum_m \frac{\partial\rho^*}{\partial t_m}\frac{\partial t_m}{\partial t_{\mathfrak{N}}}\right)$, means an improvement of utility through an increase in the public goods consumption caused by capital inflow.

Thus, equation (11) shows that a small country is better off under the tax coordination; that is, small countries have an incentive to collude with all other small countries to raise their tax rate. Moreover, the utility of large countries also improves, since large countries optimally adjust their tax rates to the tax change of the coalition and enjoy a higher tax rate than the initial level of tax rate at the non-cooperative Nash equilibrium.

This welfare effect of tax coordination among small countries is shown in Figure 2. Let $\mathbf{t}_{\mathfrak{N}}$ and $\mathbf{t}_m$ be the vectors of tax rates chosen by small countries and large countries, respectively; the equilibrium common tax rate is defined as $t_{\mathfrak{N}}^* = \arg\max_{t_{\mathfrak{N}}} \sum_{n\in\mathfrak{N}} u_n\left(t_{\mathfrak{N}}; \mathbf{t}_m\right)$ for each $n \in \mathfrak{N}$. The reaction function of the small country in its coordination is represented as $t_{\mathfrak{N}} = \gamma_{\mathfrak{N}}(\mathbf{t}_m)$. Indicating the tax rate at the non-cooperative Nash equilibrium as $t_n^* = \arg\max_{t_n} u_n\left(t_n; \mathbf{t}_i, \mathbf{t}_m\right)$ for each $n \in \mathfrak{N}$, where $\mathbf{t}_i$ is the vector of the tax rates of small countries except country n and $\mathbf{t}_m$ is the vector of the tax rates of large countries, we perceive that $\gamma_{\mathfrak{N}}(\mathbf{t}_m)$ differs from $\beta_n(\mathbf{t}_i, \mathbf{t}_m)$, which is the reaction function of country n in the non-cooperative situation.

Since the proportion of the coalition's population is denoted as $s_{\mathfrak{N}} = \sum_{n \in \mathfrak{N}} s_n$, it is recognized that $\gamma_{\mathfrak{N}}(\mathbf{t}_m)$ lies above $\beta_n(\mathbf{t}_i, \mathbf{t}_m)$[7].

On the other hand, the reaction function of a large country in this situation denoted as $\beta_m(\mathbf{t}_j, \mathbf{t}_{\mathfrak{N}})$, where $\mathbf{t}_j$ is the vector of the tax rates of the other large countries, co-incides with that in the non-cooperative situation.

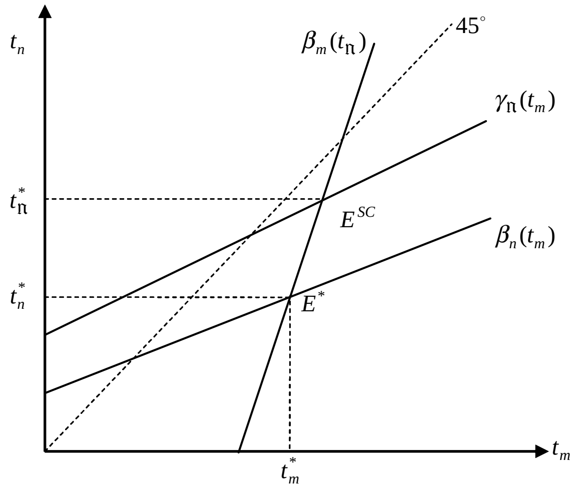

Figure 2: Small countries' coordination

Summarizing the above discussion, we obtain the following proposition.

Proposition 3 *If the reaction functions are characterized by assumption 2, the tax coordination among all small countries improves not only the utility of small countries in the coalition but also that of large countries outside the coalition.*

4.2 Large countries' coordination

Contrary to the first type of coordination, we next consider that all M large countries collude by co-operatively choosing a common tax rate but react non-cooperatively to a small country outside the coalition, while small countries behave non-cooperatively. In the same manner as in a previous case, let $\mathfrak{M}$ be the set of large countries in the coalition.

The members of the coalition choose a common tax rate ($t_{\mathfrak{M}}$) in order to maximize joint utility of the members. A small country simultaneously chooses its tax rate, taking $t_{\mathfrak{M}}$ as

[7] How the slope of $\gamma_{\mathfrak{N}}(\mathbf{t}_m)$ differs from that of $\beta_n(\mathbf{t}_i, \mathbf{t}_m)$ depends on the magnitude of the effect on ρ.

given. Now $t_{\mathfrak{M}}$ is defined as $t_{\mathfrak{M}} \equiv t_1 =, ..., = t_M$. In the following equation, t_j and t_n represent the tax rates of the other large and small countries, respectively.

We can describe the change of a large country's utility with the marginal rise in a common tax rate as follows.

$$\begin{aligned}\frac{du_m}{dt_{\mathfrak{M}}} &= (u_g - u_x)\, k_m + u_g t_m \left(\frac{\partial k_m}{\partial t_m} + \sum_{j \in \mathfrak{M}} \frac{\partial k_m}{\partial t_j} + \sum_n \frac{\partial k_m}{\partial t_n} \frac{\partial t_n}{\partial t_{\mathfrak{M}}} \right) \\ &\quad - u_x \left(k_m - \overline{k}\right) \left(\frac{\partial \rho}{\partial t_{\mathfrak{M}}} + \sum_n \frac{\partial \rho}{\partial t_n} \frac{\partial t_n}{\partial t_{\mathfrak{M}}} \right), j (\neq m) \in \mathfrak{M}.\end{aligned} \tag{12}$$

$\frac{\partial t_n}{\partial t_{\mathfrak{M}}} \left(\equiv \sum_{m \in \mathfrak{M}} \frac{\partial t_n}{\partial t_m} \right)$ is the small country's reaction to the tax change in large countries manipulating a common tax rate, and $\frac{\partial \rho}{\partial t_{\mathfrak{M}}} \left(\equiv \sum_{m \in \mathfrak{M}} \frac{\partial \rho}{\partial t_m} \right)$ is the total effect of the tax change in large countries on the net return of capital.

The three terms on the RHS of equation (12) also show the three effects mentioned in the previous case. However, the meaning of the third term is different from that of equation (10). Since the large country is a capital exporter, that is, $(k_m - \overline{k})$ is negative, the third term shows that private goods consumption decreases due to the reduction in the net return of capital.

Again, evaluating equation (12) at the non-cooperative Nash equilibrium, we derive

$$\frac{du_m}{dt_{\mathfrak{M}}} = \left[-u_x \left(k_m^* - \overline{k}\right) + u_g t_m^* k_m'\right] \left(\sum_{j \in \mathfrak{M}} \frac{\partial \rho^*}{\partial t_j} + \sum \frac{\partial \rho^*}{\partial t_n} \frac{\partial t_n}{\partial t_{\mathfrak{M}}} \right). \tag{13}$$

In equation (13), the first part on the RHS, denoted by $\left[-u_x \left(k_m^* - \overline{k}\right)\right] \left(\sum_{j \in \mathfrak{M}} \frac{\partial \rho^*}{\partial t_j} + \sum \frac{\partial \rho^*}{\partial t_n} \frac{\partial t_n}{\partial t_{\mathfrak{M}}} \right)$ means that the utility of large country is worsened by a decrease in the private goods consumption due to a reduction in the payment for capital.

On the other hand, the second part, denoted by $\left[u_g t_m^* k_m'\right] \left(\sum_{j \in \mathfrak{M}} \frac{\partial \rho^*}{\partial t_j} + \sum \frac{\partial \rho^*}{\partial t_n} \frac{\partial t_n}{\partial t_{\mathfrak{M}}} \right)$, means the utility is improved through an increase in public goods consumption.

Therefore, the sign of equation (13) depends on the degree of under-provision of public goods in a large country. For example, if residents in a large country strongly prefer public goods over private goods, the effect denoted by the second part on the RHS may exceed the impact of a decrease in private goods consumption. However, this is ambiguous in our model. Contrary to the welfare effect on a large country, small countries will be better off by tax coordination among large countries.

Similar to a previous case, the equilibrium common tax rate is defined as $t^*_{\mathfrak{M}} = \arg\max_{t_{\mathfrak{M}}} \sum_{m \in \mathfrak{M}} u_m (t_{\mathfrak{M}}; \mathbf{t}_n)$ for each $m \in \mathfrak{M}$. The reaction function of the large country in its coordination is represented as $t_{\mathfrak{M}} = \gamma_{\mathfrak{M}}(\mathbf{t}_n)$. The consequence of this coordination is shown in Figure 3. Indicating the tax rate at the non-cooperative Nash equilibrium as $t^*_m = \arg\max_{t_m} u_m (t_m; \mathbf{t}_j, \mathbf{t}_n)$ for each $m \in \mathfrak{M}$, where $\mathbf{t}_j$ is the vector of the tax rates of other large countries, we know that $\gamma_{\mathfrak{M}}(\mathbf{t}_n)$ differs from the reaction function $(\beta_m(\mathbf{t}_j, \mathbf{t}_n))$ in the non-cooperative tax competition.

Since the proportion of the coalition population is denoted as $s_{\mathfrak{M}} = \sum_{m \in \mathfrak{M}} s_m$, it is recognized that $\gamma_{\mathfrak{M}}(\mathbf{t}_n)$ lies on the right side of $\beta_m(\mathbf{t}_j, \mathbf{t}_n)$.

On the other hand, the reaction function of the small country in this situation co-incides with that in the non-cooperative tax competition.

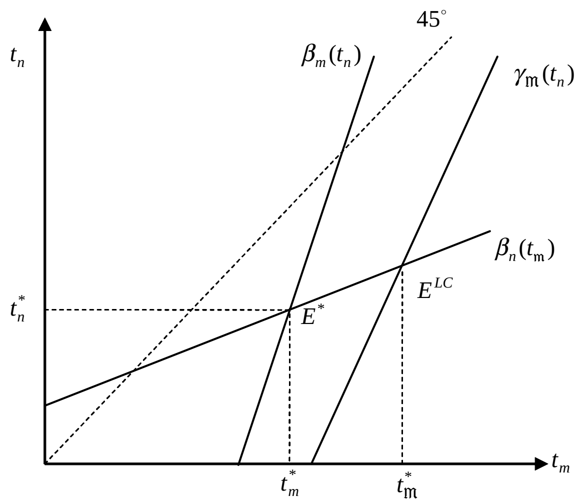

Figure 3: Large countries' coordination

From the above discussion, we derive the following proposition.

Proposition 4 *Even if the reaction functions are characterized by assumption 2, it is not obvious that tax coordination among all large countries improves the utility of all countries. Particularly, whether the utility of a large country is improved depends on the degree of under-provision of public goods.*

5 Numerical examination

In the previous section, we have implicitly assumed coalition stability. Of course, in a real world, we can perceive various rules to support policy coordination among the members of a federation. However, it must be verified whether the coalition stability is satisfied in our model. Therefore, we examine a numerical analysis about the condition for the coalition stability with some specifications.

5.1 Specified model

We assume the following three cases: first, there are two large countries and three small countries (case 1); second, there are five small countries (case 2); third, there are ten small countries (case 3). The sets of countries are defined as follows.

Case 1: $\mathfrak{M} = \{1, 2\}, \quad \mathfrak{N} = \{3, 4, 5\}$.

Case 2: $\mathfrak{M} = \{1, 2\}, \quad \mathfrak{N} = \{3, 4, 5, 6, 7\}$.

Case 3: $\mathfrak{M} = \{1, 2\}, \quad \mathfrak{N} = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

The proportion of large countries is exogenously given and that of small countries is defined as $s_n = \dfrac{1 - 2s_m}{N}$. In the following examination, we assume that country 3 chooses whether it deviates from or co-operates with other small countries, and compare its welfare levels under deviation with the coordination in each case.

The production function is a quadratic function expressed as $f(k) = 2k - \dfrac{1}{2}k^2$, $k \leqslant 2$. The initial endowment of capital $\overline{k}$ is equal to 1. Equations (3), (4) and (5) are rewritten as $\dfrac{\partial \rho}{\partial t_h} = -s_h$, $\dfrac{\partial k_h}{\partial t_h} = (s_h - 1)$, $\dfrac{\partial k_h}{\partial t_{-h}} = s_{-h}$ in a non-cooperative Nash situation, and as $\dfrac{\partial \rho}{\partial t_{\mathfrak{N}}} = -\sum_n s_n$, $\dfrac{\partial k_n}{\partial t_{\mathfrak{N}}} = (\sum_n s_n - 1)$ in the coordination case in which all small countries collude.

Utility is a linear function as $u(x, g) = x + \dfrac{3}{2}g$. For simplicity and to focus on the coalition stability, we allow that the Pareto efficient level of public goods is a corner solution.

5.2 Examination results

In each case, we assume the following three situations: (1) $s_m = \frac{1}{4}$, (2) $s_m = \frac{1}{3}$ and (3) $s_m = \frac{2}{5}$. In the first situation, if the proportion of population of the small countries coalition ($s_{\mathfrak{N}} = \frac{1}{2}$) exceeds that of the large country, the former would be worse off by coordination because of the capital outflow from the coalition to large countries. In the second situation, $s_{\mathfrak{N}} = s_m = \frac{1}{3}$, the equilibrium with the coordination is equivalent to the non-cooperative Nash equilibrium with three symmetric large countries. In the third situation, the proportion of the large country is sufficiently larger than the small country; thus, the motivation of deviation would be high.

Case 1

Table 1 shows the results of case 1 in which there are two large countries and three small countries. Each column shows the welfare levels of large and small countries at non-cooperative Nash equilibrium, coordination by all small countries and deviation by country 3 in each situation of large countries' population. The values in parentheses are the tax rates in each situation.

The differences of tax rates and utilities between large and small countries are verified in all situations. All countries are better off by tax coordination among small countries irrespective of joining the coalition. We perceive that the coalition improves the welfare of a small country even if its proportion exceeds that of one large country. Country 3 has no incentive to deviate from the coalition, since the welfare level under deviation (e.g. 1.742 in the second situation) is lower than that under coordination (e.g. 1.750 in the same situation). The coalition stability is guaranteed.

Table 1. Two large countries and three small countries

	$s_m = \frac{1}{4}$, $s_n = \frac{1}{6}$	$s_m = \frac{1}{3}$, $s_n = \frac{1}{9}$	$s_m = \frac{2}{5}$, $s_n = \frac{1}{15}$
Non-cooperative Nash equilibrium			
large country	1.709 (0.435)	1.721 (0.478)	1.744 (0.530)
small country	1.712 (0.407)	1.732 (0.399)	1.767 (0.399)
Coordination by all small countries			
large country	1.782 (0.481)	1.750 (0.5)	1.756 (0.539)
small country	1.748 (0.593)	1.750 (0.5)	1.776 (0.456)
Deviation by country 3			
large country	1.728 (0.448)	1.729 (0.485)	1.748 (0.533)
country 3	1.732 (0.419)	1.742 (0.404)	1.772 (0.401)
other small country in a coalition	1.721 (0.479)	1.737 (0.443)	1.770 (0.426)

Case 2

From table 2 that shows the results of case 2 in which two large countries and five small countries, we obtain similar implications with that in case 1. Therefore, the coalition stability is also guaranteed in this case. Tax coordination of small countries has an effective power to improve the welfare of all countries.

Table 2. Two large countries and five small countries

	$s_m = \frac{1}{4}$, $s_n = \frac{1}{10}$	$s_m = \frac{1}{3}$, $s_n = \frac{1}{15}$	$s_m = \frac{2}{5}$, $s_n = \frac{1}{25}$
Non-cooperative Nash equilibrium			
large country	1.699 (0.429)	1.716 (0.474)	1.742 (0.528)
small country	1.704 (0.381)	1.728 (0.382)	1.765 (0.389)
Coordination by all small countries			
large country	1.782 (0.481)	1.750 (0.5)	1.756 (0.539)
small country	1.748 (0.593)	1.750 (0.5)	1.776 (0.456)
Deviation by country 3			
large country	1.741 (0.456)	1.735 (0.489)	1.750 (0.534)
country 3	1.747 (0.402)	1.749 (0.391)	1.774 (0.392)
other small country in a coalition	1.727 (0.515)	1.740 (0.463)	1.771 (0.437)

Case 3

The results of case 3 which are summarized in tabe 3 are different from that of the others. In this case, the welfare level of country 3 under deviation (e.g. 1.756 in the second situation) is higher than that under coordination (e.g. 1.750 in the same situation). Therefore, it is desirable for country 3 to deviate from the coalition of small countries to co-ordinate its tax rates. Since every small country has the same incentive as country 3, the coalition stability is not guaranteed in this case.

Table 3. Two large countries and ten small countries

	$s_m = \frac{1}{4}$, $s_n = \frac{1}{20}$	$s_m = \frac{1}{3}$, $s_n = \frac{1}{30}$	$s_m = \frac{2}{5}$, $s_n = \frac{1}{50}$
Non-cooperative Nash equilibrium			
large country	1.693 (0.424)	1.712 (0.472)	1.741 (0.527)
small country	1.698 (0.363)	1.725 (0.370)	1.764 (0.381)
Coordination by all small countries			
large country	1.782 (0.481)	1.750 (0.5)	1.756 (0.539)
small country	1.748 (0.593)	1.750 (0.5)	1.776 (0.456)
Deviation by country 3			
large country	1.757 (0.467)	1.741 (0.494)	1.753 (0.536)
country 3	1.765 (0.392)	1.756 (0.383)	1.777 (0.386)
other small country in a coalition	1.735 (0.549)	1.744 (0.480)	1.773 (0.446)

We found that the coalition stability is guaranteed in the case of a few small countries. On the other hand, in the case of many small countries (e.g. case 3 of our examination), the co-operative behaviour is dominated by deviation. Thus, we expect that the threshold that decides whether the coalition stability is guaranteed exists between the cases from 5 to 10 small countries in our examination.

We assume that only one small country chooses to deviate from or co-operate with the coalition in our examination. Even if the number of deviating countries were to increase, it is not expected that the results would change dramatically due to the reduction in the benefit of deviation.

6 Concluding Remarks

In the two-country asymmetric tax competition model described in Wilson (1991) and Bucovetsky (1991), it is impossible for both countries to harmonize their tax rates to the same level due to a lack of incentive to harmonize, particularly in a small country.

On the other hand, it is shown that there is a possibility of Pareto improving tax coordination in multilateral asymmetric tax competition. We determined that partial tax coordination among small countries can improve the welfare of all countries engaged in tax competition, irrespective of participating in a coalition. The reason is that a coalition can internalize fiscal externality effect. Furthermore, we found that coalition stability is satisfied in the case where there are not many small countries in a specified model. The case in which all small countries collude to raise their common tax rate is different from the minimum tax scheme considered in Peralta and van Ypersele (2006) in that a tax rate is chosen co-operatively.

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